

# SCHOOL SCIENCE AND MATHEMATICS

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## THE POSITION OF ATOMIC THEORY.\*

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The position of the atomic theory and its use as a device in the teaching of chemistry might well occupy our attention, for frequently we hear it said that the theory has outlived its usefulness and is no longer a necessary part of our courses. In spite of these assertions, it still finds a prominent place in our recent text-books and among many other uses to which it is put, it is still called upon, as in the days of John Dalton, to present to our minds some suitable mechanical conception for the explanation of the laws governing combination by weight and volume.

It is frequently urged that these mechanical conceptions, or theories if you will, should not be taught to our beginning students and that all such notions as the atomic theory and the theory of electrolytic dissociation should be left to more advanced stages of the work. It is urged, and with some justice, that our students immediately confound the fact with theory and unconsciously soon come to the point where, as in the case of the atomic theory, they believe that matter is actually made up of small particles called atoms. Is this the fault of the student entirely? Do we as teachers make perfectly clear the justification for the existence of any theory? Alexander Smith, in his recent magnificent book, says: "The language of chemists has become so saturated with the phraseology of the atomic and molecular hypotheses, that we speak in term of atoms and molecules as if they were objects of immediate observation." Another reason for this difficulty seems to be that the correspondence between fact and theory in many cases is apparently so complete that the former is considered a proof of the latter. In

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\*Read before the Ohio Mathematics and Science Teachers' Association Dec. 27, 1906.

reference to this point Smith says: "This fallacy is one of the commonest into which the student of science falls. This inference would be equivalent to asserting the impossibility of devising any other hypothesis which should correspond equally well with the facts." "One does not require to give up the advantages of the atomic hypothesis if one bears in mind that it is an illustration of the actual relations in the form of a suitable and easily manipulated picture, which may on no account be substituted for the actual relations." This is the testimony of Ostwald.

It has been said that the object of all science should be the elimination of hypotheses. With this we are all agreed. In reference to the whole question of theory, it must be kept uppermost in our minds that the theory is merely a convenient device by means of which one can best explain to himself the behavior and properties of matter. Some of the objections to the atomic theory arose undoubtedly at the time of the announcement, by Arrhenius, of the theory of electrolytic dissociation which gave such an impetus to research along the lines of physical chemistry. Even the most enthusiastic admirers of this most helpful conception recognize some obstacles which are still to be overcome before one can draw a complete and correct picture of the behavior of solutions. This working model simply explains to us in a convenient manner the facts, among many others, of abnormal lowering of the freezing point, an abnormal elevation of the boiling point, and abnormal osmotic-pressure phenomena. A full discussion of the question of teaching theories would form the subject of several papers and since this is not my main purpose, I shall leave the matter at this point with the few preceding remarks. It will be my attempt to point out the services which the one great central theory of the physical sciences has rendered in the development of the science of chemistry and therein, to my mind at least, rests its chief justification and glory.

As generally stated in our leading text-books the three assumptions which constitute Dalton's Atomic Theory may be briefly stated thus: "(1) All elements are made up of minute independent particles to which Dalton gave the name atoms.

"(2). All atoms of the same element have equal masses; those of different elements have different masses; in any change to which the atom is subjected its mass does not change.

"(3). When two or more elements unite to form a compound the action consists in the union of a definite small number of

atoms of each element to form a small particle of the compound. The smallest particles of a given compound are therefore exactly alike in the numbers and kinds of atoms they contain, and larger masses of the substances are simply aggregates of these particles."

In the light of the tremendous advances that are being made at present in the realms of molecular physics and physical chemistry, it might be well to turn our attention to the examination of this theory and if possible ascertain whether it still deserves to occupy the commanding position which it has held for one hundred years. To do this it might be well to take up each of its subdivisions in turn and determine in what relation each stands to our present accepted views.

As given above, the theory postulates first the existence of small discrete particles called atoms. A critical examination of this statement involves the whole question as to the ultimate constitution of matter and in the time allotted to this discussion we can recall only the principal investigations along this line. Among the earliest attempts at a satisfactory solution of this question stands what is now called "Prout's Hypothesis," enunciated in 1815. The work of Prout consisted in an examination of the relative atomic weights then known, in the hope of discovering some relationship among the elements which might serve to answer the question. His conclusion was that all elements were merely condensations of hydrogen, which substance was considered the primal element. This last statement may be considered the final view of Prout and a modification of his earlier propositions. In his own words: "If the views we have ventured to advance be correct, we may almost consider the first matter of the ancients to be realized in hydrogen; an opinion by-the-by, not altogether new." Gmelin, in a study of the atomic weights, made this statement: "It is surprising that in the case of many substances the combining weight is an integral multiple of that of hydrogen, and it may be a law of nature that the combining weights of all substances can be evenly divided by that of the smallest of them all." The hypothesis of Prout proved to be a great stimulus to more accurate atomic weight determinations in the hope of finding some exact experimental evidence for its justification. In spite of the masterly work of Berzelius and later that of Stas, some modifications were attempted in which the chief idea was that a fractional part of the hydrogen atom might be the primal element. Venable sums

up the attempts in this direction as follows: "Any fraction whatsoever of the weight of the hydrogen atom cannot be considered without abandoning the fundamental idea of an atom and any such consideration can have no clear meaning nor be given any true value." Recently, there have been some attempts to attach considerable importance to the hypothesis of Prout. This has arisen from the fact that when the relative atomic weights are calculated on the basis of oxygen being 16, it is found that about 23 are whole numbers and 17 are approximately so. In reference to this Jones remarks in his recent book: "Taking all of the facts into account, we recognize, of course, that the hypothesis of Prout, either as originally proposed, or as subsequently modified, is not rigidly true, but we still feel intuitively that there is something in it. The coincidences are far too numerous to be attributed to mere chance." Dr. F. W. Clarke, in the Wilde Lecture delivered before the Manchester Literary and Philosophical Society in 1903, says: "Prout's hypothesis is discredited, and yet it may prove to be a crude first approximation to some deeper truth as the probability calculations of Mallet and Strutt would seem to indicate. The approaches of the atomic weights to whole numbers are far too close and too frequent to be regarded as purely accidental."

On an historical examination of the development of chemistry following the idea of Prout, one sees standing out prominently the forerunners of the Periodic Law. Notable among these were Doebereiner's Triads in which a study of such like elements as chlorine, bromine, and iodine revealed that bromine was the arithmetical mean of the other two in its physical and chemical properties. This led the way to the Law of Octaves announced by Newlands in which it was shown that there was a constant recurrence of properties when the elements were arranged according to their atomic weights. The culmination of all these studies into the search after a relationship between the elements from a consideration of their relative atomic weights resulted in the Periodic Table proposed by both Lothar Meyer and Mendeleeff. Immediately Mendeleeff predicted the properties of eka-boron, eka-aluminum, and eka-silicon and to-day\* he has the gratification of knowing that the properties of gallium, scandium, and germanium are almost identical with those of which he had foretold. Thus we have placed in our hands a

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\*Mendeleeff died Feb. 2, 1907.

device which at once simplifies our study, predicts new facts and corrects many of our physical constants. Of recent benefit, one need but recall the work in reference to the atomic weight of radium and its final disposition to a place in the table\* in the same group as its closely related substance, barium, in order to appreciate the value of this field of endeavor. In concluding this paragraph listen to Mendeleeff's own words: "The law I announced has been considered a repetition in another form of what has been already said by others. It is now certain that the Periodic Law offers consequences that the old systems had scarcely ventured to foresee. Formerly it was only a grouping, a scheme, a subordination to a given fact; while the Periodic Law furnishes the facts and tends to strengthen the philosophic question which brings to light the mysterious nature of the elements. This tendency is of the same category as Prout's Law, with the essential difference that Prout's Law is arithmetical and that the Periodic Law exhausts itself in connecting the mechanical and philosophical laws which form the character and glory of the exact sciences. It proclaims loudly that the nature of the elements depends above all on their mass, and it considers this function as periodic. The formula of the law might be changed, a greater appreciation of this function will be found, but I believe that the original idea of the Periodic Law will remain."

In 1867 Lord Kelvin was led to propose the vortex theory of the atom. In reference to this theory Risteen remarks: "That it is full of enormous mathematical difficulties and that for this reason we can regard it, at present, only as a highly interesting possibility, whose consequences must be traced out by future generations." It lacked value in that it could not serve as a simple "working device" owing to its extreme complexity and herein should lie the strong point of any scientific conception. In 1878 Lockyer advanced the belief that the so-called elements were compound in their character. He based this on spectroscopic evidence adduced in the field of astronomy. His work received very great favorable attention at the hands of Crookes and its influence is clearly manifest in the subsequent papers of the latter.

For some years now the scientific world has had its attention fixed upon the wonderful researches that are being made in the

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\*See Ramsay: *Science* August 2, 1907, page 158.

fields of radio-activity, and also in the studies upon the conduction of electricity through gases. From the large mass of experimental data on hand one can merely point out some of the most important steps and deductions in reference to their bearing on the ultimate constitution of matter. To do justice to a review of this field of scientific research, one would of necessity be compelled to confine himself entirely to an examination of some of the most fascinating investigations of recent years. For our purpose, we may sum up a few of the main points as follows:

(1). It has been found that gases become conductors of electricity under the following conditions:

a. At high temperatures or when in contact with glowing metals or carbon.

b. When subjected to the influence of the cathode rays, Roentgen rays, ultra-violet light, and radio-active substances.

c. When diffused through space through which an electric charge is passing.

(2). It has been shown that the phenomenon of the conduction of electricity through gases is due to the presence of gaseous ions, some being charged positively and some charged negatively.

(3). J. J. Thompson has shown by a study of these gaseous ions that the charge on the negative ion is the same as that carried by a univalent ion in solution. We must keep this point firmly fixed in our minds as it has bearing on our next step. Since it has been shown that the value of the ratio "charge divided by mass" ( $e/m$ ) for the negative gaseous ion is  $1 \times 10^8$  and that of the univalent ion hydrogen in solution is  $1 \times 10^4$ , it is clearly evident then that the mass of the hydrogen ion in solution must be 1,000 times greater than that of the negative gaseous ion. Furthermore, since the theory of electrolytic dissociation defines the hydrogen ion as the atom carrying an electric charge, we find therefore, that the negative gaseous ion has a mass approximately 1-1,000 that of the hydrogen atom. More accurate determinations have shown this figure to be 1-770. To these small particles, Thomson gave the name "corpuscle."

(4). When the value of the ratio "charge divided by mass" ( $e/m$ ) is determined for the particles shot off by the cathode rays and the so-called Beta rays of such radio-active substances as radium, it is found to be identical with the value determined for the negative gaseous ion or the corpuscle. Furthermore, the corpuscles and the Beta rays are alike in the following in-

teresting properties: They are deflected by the magnet, give rise to heat in bodies which they strike and they act as nuclei about which atoms and molecules collect. Regardless of their source, it has been found that the value of  $e/m$  for the corpuscles or electrons is practically a constant.

Here we have then what seems to be the first tangible experimental evidence for the existence of particles of matter which are sub-atomic in their character. In 1879 Crookes was led to say that "in the phenomena of the vacuum tube at high exhaustion, the particles constituting the cathode stream are not solid, not liquid, do not consist of atoms propelled through the tube—but that they consist of something much smaller than the atom—things which seem to be the foundation stones of which atoms are composed." Based upon the great mass of experimental data in this field as to the existence of the corpuscle, Thomson in 1904, by means of an elaborate mathematical analysis offered his highly interesting speculation regarding the structure of the atom. Briefly described in his own words it is as follows: "We suppose that the atom consists of a number of corpuscles moving about in a sphere of uniform positive electrification; the problems we have to solve are: (1) what would be the structure of the atom, i. e., how would corpuscles arrange themselves in the sphere? and (2) what properties would this structure confer on the atom?" His solution to the first problem is, that the corpuscles arrange themselves in a series of concentric rings if constrained to move in one plane. From this highly interesting conception an interesting explanation was worked out for the periodic table. Likewise some of the other properties, such as valence, find an explanation by means of Thomson's idea. It is thus seen that at present the trend of scientific investigation seems to point to masses which are sub-atomic in character.

Dalton's Atomic Theory postulates in the second place that the atoms of the same element have equal masses and in any change to which an atom is subjected its mass does not change. A critical review of chemical thought in this direction would be a history of the development of more refined methods of work in the laboratory. In 1886 Crookes was led to remark: "It may be well questioned whether there is absolute uniformity in the mass of every ultimate atom of the same chemical element. Probably our atomic weights represent a mean value around which the actual atomic weights of the atoms vary within certain limits."

In June of this year there appeared an article by Landolt in which it was shown that after a very careful study of some well chosen chemical changes discrepancies in weight existed which exceeded the limits of experimental error. Data on this phase of the theory do not justify any modification of the same and a discussion here would involve problems not intimately connected with our main purpose.

It is that portion of Dalton's theory which refers to a compound as the result of the union of definite small numbers of the atoms of elements to form a small particle of a compound that has been of such great benefit to chemists. In fact the first stage of the development of chemical theory occupied itself with the idea of constancy of composition as a characteristic of a chemical substance. The predominating idea of the second stage was that chemical individuality of a substance depended on its constitution. To Liebig, Wöhler, and Gay-Lussac we are indebted for the germ of isomerism. The introduction in 1850 of the valence conception served to clear up a great deal of confusion existing at that time and to bring together many of the facts of chemistry into one harmonious whole. As a debt that the scientific world owes these extensions of the doctrine of atoms one need but to refer to the master idea of Kekulé, which led him to propose his structure of the benzene ring and the enormous interest this gave to the study of carbon chemistry. For some time it was believed that the method of writing structural formulae would have to be abandoned owing to the discovery by Wislizenus of the isomerism in the lactic acid series. As a result of this there came into being the conception of the asymmetric carbon atom, by means of which structural formulae, possessed the possibility of becoming tri-dimensional. For this modification we are indebted to Vant Hoff and Le Bel, who imagined the carbon atom as occupying the center of a tetrahedron at each apex of which was an affinity unit. Thus they were able to explain the troublesome tartaric acid chemistry. As an extension of their ideas we need but to refer to the work being done at present by Pope of England and Widekind of Germany on the optical isomers of quinquivalent nitrogen.

In recent years the field of organic chemistry has been witnessing an interesting struggle in the attempts of investigators to explain the dual chemical nature of certain organic compounds. Here again the doctrine of atoms by a simple extension offers us a ready explanation in the conception of tauto-

merism, i. e., a shifting of a hydrogen atom, say, from one carbon atom to another. Such an idea in connection with the well understood principle of chemical equilibrium best explains the chemistry of such bodies as aceto-acetic-ether, and phloroglucin.

Until recently carbon monoxide was the only compound known in which the carbon atom was bivalent. From the brilliant researches of Nef, of the University of Chicago, we know that the behavior of many series of organic substances is best explained by the assumption of the bivalent carbon atom. One need but recall his classic work on fulminic acid, prussic acid, and the iso-cyanides.

In the same manner one might bring forward one example after another in which the atomic theory has been of such untold service. It undoubtedly reaches the climax of its strength in this field of molecular constitution and its various consequences. I believe that it will still continue to maintain its usefulness in this direction with ever increasing force and power.

Finally then, we observe that the trend of modern investigation seems to point to sub-atomic masses. The facts of chemistry, however, show that in all chemical changes which take place the atom seems to be the unit of transfer. For this reason, I believe that the so-called atomic weight will still continue to give us the best explanation of the law of constant composition and the law of multiple proportion. At present there are no facts of chemistry which need the extension or replacement of Dalton's idea in order to best explain these points. In reference to the corpuscular theory of the atom, it might be added that the question has already been asked as to the ultimate nature of the electron itself; i. e., is matter electrical in its nature?

So, in the present state of our knowledge, Dalton's theory still continues to offer the simplest and most ready conception of the main facts of chemistry. Our final judgment on such theories as those of Lord Kelvin and J. J. Thomson must be suspended, at least for the present. As to the teaching of Dalton's theory or any other theory let us keep ever before the minds of our students (and I may add, ourselves as well) that the theory itself is only the scaffolding from which our edifice is built and is no integral part of the structure itself.

As to the usefulness of theories in general to the investigator and Dalton's atomic theory in particular to the chemist, let us once more remind ourselves of the history of the development of

chemical and physical theory. "Theories are necessary for further development, and that, though the actual teachings of science may lie in the facts, the real intellectual significance can only be acquired by connecting isolated facts by means of hypotheses." Other problems will present themselves for solution in the future and these will demand some ready and convenient explanation. For this reason we will undoubtedly still witness further extensions of this useful idea as times goes on, and I firmly believe that it will be the light which shall be the guide for investigators to even more brilliant achievements than in the past.

As an apology for an examination of this general subject let me quote from President Rückers' address before the British Association: "It is well to fix our attention on some of the hypotheses and assumptions on which the fabric of modern theoretical science has been built, and to inquire whether the foundations have been so 'well and truly' laid that they may be trusted to sustain the mighty superstructure which is being raised upon them."

I wish to close with a brief paragraph from Dr. Clarke's splendid paper already referred to: "If we take the atomic theory out of chemistry we shall have left but a dust heap of unrelated facts. The convergence of the testimony is remarkable and when we add to the chemical evidence that which is offered by physics the theory becomes overwhelmingly strong. And yet, from time to time, we are told that the theory has outlived its usefulness and that it is now a hindrance rather than a help to science. When we say that matter as we know it, behaves as if it were made up of very small, discrete particles, we do not lose ourselves in metaphysics and we have a definite conception which can be applied to the correlation of evidence and the solution of problems. Objections count for nothing against it until something better is offered in its stead, a condition which the critics of the atomic theory have so far failed to fulfill."

## CHEMISTRY SYMPOSIUM.

- I. *What to Put in a Notebook.*
- II. *When and Where Should the Notebook be Written Up.*
- III. *Corrections of Notebooks.*

## THE NOTEBOOK.

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*What to put in the Notebook* largely depends upon the aim of the course in laboratory work and the methods employed in carrying out that aim.

Besides the usual benefits derived from laboratory work—skill in manipulation, training in observation, and reasoning, etc.—I believe that it is *most* useful when it also assists in making the text book interesting and clear. The pupil remembers but few of the facts of the book and fails to get the slightest idea of the theoretical parts unless enlightened by the work in the laboratory. The beginner in chemistry approaches a subject entirely out of his experience. The laboratory work is necessary to build up an experience before the text is really understood.

After the experience is acquired, it is necessary that he should see the relation of this to the related subject matter in this text. It is best that the pupil study out this relation for himself, and this is largely possible if he is properly directed. The easiest way the teacher can determine whether the pupil understands this connection is through the Notebook. Therefore besides the things which must necessarily be embodied in the notes, i. e., what is done and what is observed, I wish the notes to show what bearing the experiment has on the related topics in the book.

It often happens that the experiment is related to something in the pupil's experience not included in the text. I think it a good thing to have this included in the notes so that it may be brought out again when a recitation or review is made.

*In writing up the Notebook* we try to eliminate everything possible, yet make the notes complete enough to furnish a thorough review of the work in the laboratory when such a review is desired.

While I do not favor any particular *hard and fast* order of

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\*Read before the conference of Chemistry, a section of the Michigan Schoolmasters' Club, March 28, 1907.

topics in writing the notes, we usually use the following heads: number, date, name, material, operations and results, and discussion. Under operations and results are included:

Drawings, if any:

What was done and what observed; and equations, if any. The drawings are not elaborate but merely diagrammatic with only the essential parts to the apparatus. We make the drawing such that the arrangement of the apparatus, the chemicals used, whether heat was necessary or not, and the products obtained may be seen at a glance.

I do not consider it necessary to copy the whole of the directions to an experiment, but something more than the mere results is needed so we include both under one head and write the notes such that the directions and results will both be evident in the same sentence or sentences.

The following record of an experiment taken from a pupil's notebook will illustrate what I mean on that point:

**"A Chemical Compound.**

"Operations and results.

Sept. 18, 1906.

"(a). Thoroughly mix equal amounts of iron and sulphur in a mortar. The resulting color is light green. If a magnet is passed over the mixture it will remove the iron. The sulphur is moist and therefore some of it adheres to the iron. Again thoroughly mix the two and put a little carbon disulphide on some of it in a test tube. Stir and pour a little of the liquid on a glass. After the carbon disulphide has evaporated, sulphur will be left, which shows that the sulphur is soluble.

"This mixture of iron and sulphur is a mechanical mixture.

"(b). Place a little of the mixture in a test tube and heat. The mixture begins to glow and continues to glow even if taken from the flame. The burned substance is black. If a magnet is passed over the substance it will remove a very small amount of the iron, and if carbon disulphide is added it will remove a very small amount of sulphur."

The following, taken from the same experiment, illustrates what we included under the *discussion* in this particular experiment:

"When the mixture is heated nearly all of the iron and sulphur disappears and forms a new substance which is called a chemical compound. In the chemical compound the identity of the sulphur and iron is lost. In this experiment it is shown that heat

will change some mixtures into chemical compounds. The class of chemical action which here took place was synthesis or combination."

As was said before, what was included under the discussion was determined by the pupil through proper questions. If this cannot be done, as is the case in some instances, I state outright what I want them to see.

We do all notebook work in the laboratory. At the time of or just after doing the experiment and before another is started. The pupils are required to present the experiment or notes for acceptance as soon as written. The experiment is fresh in the mind of the pupil. You have a good chance to ask questions relating to the experiment and if anything is to be added or corrections to be made, then is the best time to make them. If the experiment needs to be repeated it can be done then with the greatest economy of time.

*Corrections.* Sometimes the pupils add the corrections to the end of the experiment. But where they are slight they are made in the body of the notes. If a number are necessary so as to mar the looks of the book the experiment is rewritten.

No experiment is accepted until it is complete in every respect. After that I have nothing further to do with the Notebook. We complete each section of work in both laboratory and text before proceeding to the next. After a recitation on the whole we have no further use for the Notebook except for reference.

I try to make the Notebook an affair of the pupils and thus relieve the teacher of the care.

From the above it is evident that the Notebook is simply a *means* and not an *end*. It is a means of bringing laboratory work and text-book work into connection and is therefore of secondary importance. We do our laboratory work not to make a Notebook but to gain an experience that will enlighten and enliven the text.

BY H. S. REED,

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Before beginning the discussion of the subject assigned to me I must be truthful and say that it is not exactly to my liking. It savors of that branch of pedagogics which I have never been quick to defend. I believe that good teachers are born, not made. I do not mean by this poor teachers cannot be made

better, or even excellent teachers improved by training, but I do not believe that any amount of training can make a successful teacher if he has no natural ability back of it. I hold this same opinion with reference to other professions, and am a great believer in the old adage, "Many a good carpenter is spoiled to make a poor preacher." These brief remarks may seem somewhat irrelevant to my subject, but the idea I wish to bring out is this: the success of the teacher, and by the success of the teacher I mean the welfare of the student, depends infinitely so much more on his personality than on his method that no fixed rules for keeping notebooks can be applied to all teachers. Their different natural manners of presenting the subject may and must, to be efficient, require different forms of notebooks.

My ideas concerning notebooks are the natural outgrowth of my teaching. I present my division of chemistry to the students of the Agricultural College by means of lectures and laboratory work, requiring notebooks in each. I also assign reading lessons in a text-book and advise the student to purchase one, but leave that matter optional with himself.

At the suggestion of Mr. Peet I have considered the subject under three heads, the first of which is. "What to put in a notebook?" In answer to that question I should say anything and everything the student gets from his teacher if it is relative to the subject. I do not believe in "set form" notebooks leaving simply blanks to be filled in by the students. Notebooks of this description take away the student's imagination, destroy his power of initiation, curb his originality. I don't know how the most of my hearers feel under the same circumstances, but when I stand up before a hundred or so students assembled to hear my lectures in organic chemistry it is not my object to make chemists out of them—ninety-five per cent probably will never do anything with chemistry after graduation—but to give them a scientific knowledge and with the aid of the laboratory a scientific training that will be of practical utility to them throughout their after life.

I have here a page from a certain laboratory manual and "set form" notebook. The first word on the margin is "Object," then follows, "The preparation and study of Methane,  $\text{CH}_4$ ." I consider that to be simply one of the objects and I do not want my students to think that is all they are going to learn from the experiment. For, to the average student, the consid-

eration of the mechanical and physical principles conveyed by the apparatus, the bodily skill acquired with his hands in setting up the apparatus, the exercise of his originality and judgment in methods during the study of the compound and its preparation, the economy of this particular method in comparison with others, are objects which the student should bear in mind. The object of an experiment cannot be stated so briefly. Some may say that the preparation and study of methane covers all these points, but no student will so analyze it.

The next words appearing on the margin are "apparatus," "chemicals," "method." The paragraphs of their respective directions follow each. I do not object to printed directions being placed in a notebook but prefer that the student write the directions in his book himself—telling exactly what he did. It is excellent training in descriptive writing and tends to fix more clearly on the student's mind the work he has been doing.

Next follow short outlined experiments with blanks for results. When the student enters the laboratory he should have some knowledge of the substance with which he is to work, and I believe it is better to ask him a few questions to suggest a line of thought and let him work it out for himself, recording his results as he obtains them.

The incompleted reaction  $\text{NaC}_2\text{H}_3\text{O}_2 + \text{NaCh} = \text{CH}_4 + ?$  next appears. This suggests the mere mathematical balancing of the equation which certainly is not the right impression to give the student. And so on, with nothing of a qualitative nature, nothing but short outlined experiments illustrating chemical phenomena with blanks for results. I may be mistaken but to my mind that sort of teaching makes a very shallow impression on the student. Rather, give an outline on the preparation of Methane, ask him a few questions concerning its properties, require him to use a known quantity of sodium acetate and report to you the per cent or the theoretical yield he actually obtains, let him write up his work in a blank notebook in his own language telling exactly what he did and what his conclusions were. The training derived will be of inestimable value to the student and when you look over his notes you will have some idea of what he has been doing and thinking about.

I am against the use of "set form" notebooks also because I believe they render it difficult for the teacher to determine whether or not a student is doing individual work. Short answers filled in blanks on a specified outline must of necessity be

very similar, while if a student is writing full notes there is a certain individuality to his work which often exposes one who is copying.

I require two sets of notes, one on the lecture, the other the laboratory work. I do not allow my students to mix their notes in these two divisions of the subject. I prefer that the lecture notes be written with a margin in which the student writes a word or two suggesting what is comprehended by the following paragraph. I believe that the sentences should be completed, and while I pay no attention to either English or spelling, other than the spelling of technical terms, still I think the student should take pains in these branches for his own good and satisfaction. I do insist, however, on lots of good drawings, not artistic, but drawings that are somewhere near mechanically correct. I want a Leibig condenser drawn so that anybody not familiar with it could pick one out of the stock room on inspecting the sketch. I insist also that a student in his drawings pay particular attention to the connection of one piece of apparatus with another. For example, the sketch of an ordinary gas wash bottle must show the construction of the bottle, how the intake tube dips beneath the surface of the absorbing liquid and that is the side of the apparatus which must be connected to the gas supply. Features of this kind show that the student knows what he is doing; and I believe that in ninety-nine cases out of a hundred if a student can set up a piece of apparatus correctly you may rest assured that he will know what to put into the apparatus to accomplish the desired result. It is one thing to know that alcohol and sulphuric acid react under certain conditions to form ether; it is another to know in what kind of an apparatus to accomplish this reaction.

The time for writing up notes, in a college at least, must be arranged by the student to suit his convenience. I have no objection to a student writing up his laboratory notes during a laboratory period if it can be done without loss of time to his experimental work; and I have no objection to a student writing his lecture notes directly in the book he hands in for inspection if it could be done, but to my knowledge notes have never been so written. What I do prefer is that the student write his notes briefly in class and then copy and arrange them during a study hour carefully and neatly in a book to be handed in for inspection. That is all the studying I require of my students. If a student carefully and thoughtfully copies and arranges his notes

on the evening of the day he took a lecture or performed an experiment in the laboratory I believe he will receive an impression more lasting and satisfactory in every way than by simply studying an assigned lesson in a text-book. I am not talking discredit to text-books; I think they are very useful but also think that some ways of using them are better than others.

Some teachers at the Agricultural College have sometimes complained that our department requires too much time of the students to write up notes. I have always claimed that it takes no more time to copy and arrange notes than the student should reasonably spend on the study of the subject outside of the class room; and so long as that is the only time we ask we have not overstepped our share of demand on the students' time.

So much concerning what to put in a notebook, when and where it should be written up, and now I come to that division of my remarks which, as teachers, lies very close to our hearts; that is the correction of notebooks.

Perhaps some of you may have agreed with my ideas up to this point and said to yourselves that it is all very good but impracticable on account of the tremendous amount of work necessary to correct books of that description. I must say that I have never found that branch of my work so very hard, or tedious; in fact, for the most part I have rather enjoyed it.

I look over the lecture books myself and leave the laboratory notes to the instructor working with me in the laboratory. Both lecture and laboratory notes are not called for at the same time until the end of the term. Generally I ask for the laboratory books about two weeks after the lecture books. I look over the lecture notes the first time two or three weeks after the opening of the term, and at the next lecture period discuss before the class the failings and also the good points of their notes, making suggestions at the same time as to how they may be improved.

In grading the notebooks the first thing to which I direct my attention is the neatness. The condition of the notebook in this respect tells me what I may expect from the student on his work in general all through the term. I next make an estimate of the number of pages, not requiring, however, a certain number. I know about what to expect from experience. There is a man teaching chemistry at the University of Kansas who at one time required a certain number of pages as a criterion of excellence, and if the book had this number of pages of written matter it was accepted. Having a little extra time one day he was care-

lessly looking through some of these books and in one found seventeen pages of Ben Hur copied in the middle. I then look down the margin for headings of subjects treated in the following paragraphs and also read here and there a paragraph. These observances with a hurried inspection of the drawings give me, I believe, a fair knowledge of what the student is doing and also enable me to grade him justly on his notebook.

In this way I am able to look over about thirty notebooks an hour. On an average I have ninety books to grade four times a term, making twelve hours' work in all: not a great expenditure of time for the benefits gained.

Perhaps I had better say here the third heading of the subject under discussion might better be stated by me "looking over" or "grading" notebooks instead of "correcting" them. I make a very few corrections on the books themselves. If the book is accepted there are not many corrections to make; if not, I call the student to my office and talk over the condition of his work with him. He likes that treatment better than written corrections which often he does not understand. It gives him a chance to ask questions and he does better next time. This of course takes some of the instructor's time, but not a great deal, and it is well spent. Out of ninety books I seldom have to grade more than ten or twelve below a passing mark.

Notebooks of the type I have been recommending often have their humorous side which relieves the monotony of grading them. One young lady in my class wrote in her notebook that she got a very poor yield on her preparation of ethyl bromide because the substance boils at  $39^{\circ}\text{C}$  and she held the flask for some time in her warm hand. Another notebook contained a drawing showing the student in his room copying notes. There were many good features in the drawing, but the most impressive was a mouse climbing the leg of the study table showing the intense stillness of the room and application on the part of the student. In a notebook in cooking a young lady who was taking elementary chemistry wrote that water was not exactly a food but it helped on some. You must, however, always be careful to take it in the right proportions: two parts of hydrogen to one of oxygen.

*(To be continued.)*

**MODELING WAX IN PHYSIOLOGY AND ZOOLOGY.**

BY ERNEST C. WITHAM,

*Science Teacher, Perkins Institution for the Blind, South Boston, Mass.*

The laboratory work in physiology and zoölogy with the present standing of these subjects in the average high school is at the best limited. Besides the use of specimens and charts it was my experience when in "sighted schools," that numerous blackboard diagrams and illustrations were very helpful to partially obviate this difficulty. When I came to the Perkins Institution for the Blind the blackboard was useless, so I gradually took up model making in my classes. I still have a means of making diagrams on paper, and I use them considerably; but I find that they do not convey nearly as clear an idea of the part as models do. Perspective is a thing that I find is very difficult to make clear to the blind students by diagram.

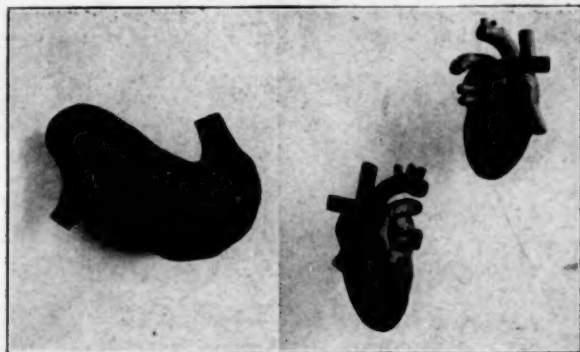


Fig. 1.

Fig. 2.

Although nearly all of my students have learned to use clay to some extent before they get to me, I do not find this substance very satisfactory. There are in the institution a large number of papier mache and plaster of paris models, but the former offer very little resistance to abrasion and the latter are very brittle and easily destroyed. So, for the rough usage that models must certainly get from blind boys, these are not all that could be desired.

The substance that I now use almost wholly is modeling wax. It is the same material that the dentists use for making impressions of the mouth. I do not claim to be the first to employ wax

for modeling, but as far as I know this substance is not generally used to-day in the schools for the purpose. My excuse for this paper is merely that I believe that I am using something to advantage in my classes that is not widely known to teachers.

This wax costs \$1.00 per lb. new. The dentist cannot, or at least should not, use this material over again for a second mouth; but this second-hand wax is just as good for my purpose and I use it altogether. It has never cost me more than fifty cents per lb. This wax is very easy to work; and makes desirable and durable models. It can be obtained from any dental supply company. If there is a dental college near by I imagine that a teacher could arrange to get some of their old wax at a very reasonable price.

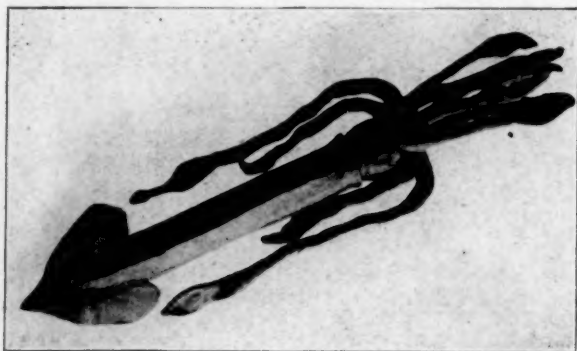


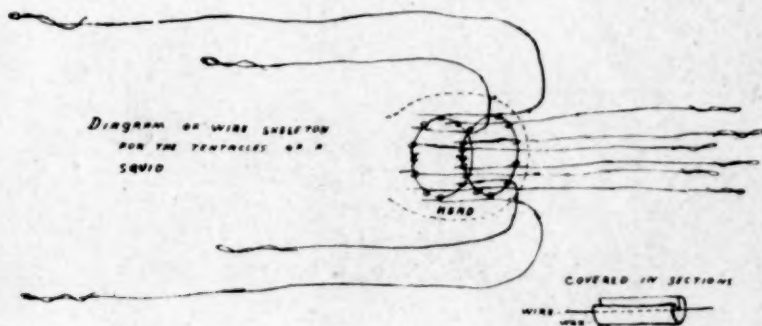
Fig. 3.

This wax softens up very readily in warm water, and can then be quickly worked by the hands into any desired form. It is best to get the general shape of the object first, then after the wax has cooled (this cooling may be hastened by cold water) so that it will not easily collapse, it can be smoothed off and slightly altered by just holding it in the warm water long enough to soften up the outside.

If the model is to be a complicated one, it is best to make the various parts separately and then join them together one at a time. For this purpose my students use a small stream of hot water from the faucet. Thus they heat only the two surfaces that they wish to join together. This joint is thoroughly cooled before any other additions are made. Working carefully in this way, allowing the hot water to flow over only the part that they wish to join or alter, my students keep their specimens from collapsing.

If there are to be many small projections, as for example, the tentacles of any of the cephalopoda, it is best to make a skeleton of wire, or of some other material, for the wax. A small cord was used for a snake. Also, where there is a large volume, to save material, my scholars sometimes use a block of wood for filling which they cover with the wax.

Another desirable feature about this wax is that it may be cut very easily with a knife if it is slightly heated. Sometimes cutting away is the best means of getting a desired effect. In the cold wax one may easily carve any delicate part of a model, with good results. In this way I made the face to a model of a discus thrower worked in wax.



After all, the work in wax is so easy, that the only important thing is to acquire the knack of keeping the wax that is being used at just the right temperature so that it will not wrinkle on the outside. That is, after the general shape has been obtained, keeping the surface warmer than the interior.

There is of course no waste of material, as the wax may be used over and over again. The students prefer the wax modeling to clay modeling; and the results of their work is very encouraging. The models are durable; but if they are damaged, usually the break is in one place, and by simply holding the broken surface under the hot water faucet a few seconds, the model can be made exactly as good as new.

These models were made by blind students:

Fig. 1. Anterior and posterior views of two models of the human heart.

Fig. 2. The model of a human stomach.

Fig. 3. The model of a squid (arrowfish).

## THE EDUCATIONAL VALUE IN THE STUDY OF ANIMAL LIFE.

BY ALONZO P. FROTH,

*Spokane High School, Washington.*

An attempt is made in this article to state in a clear manner what there is in the subject-matter of zoölogy and what mental tendencies the child has that render this work capable of furnishing a certain fundamental discipline that can be obtained only with greater difficulty in other subjects.

In this discussion of the educational value of the study of animal life, I do not wish to show overstatement, nor do I claim for it an occult or exclusive value. I simply wish to state what I am trying to accomplish in teaching the work at the present time. My experience as a science teacher is limited, but not fruitless of results and of convictions. It is necessary to show how the best discipline may be obtained and, in so doing, state briefly yet as clearly as possible the method of teaching.

The pedagogical content of this work may be analyzed into the following eight purposes, or values, which should be held in mind and made to determine the method of presentation, so long as the work is to be taught for an educational value:

First: The work should be so presented that a habit of gathering original information will be formed. This is to be done by placing the pupil in touch with the source from which this "first hand" information is to be obtained. Here we see the mental process of observing predominate. Whenever and wherever this purpose is accomplished, the laboratory method is being used. That with which the teacher works at this stage is the inborn inquisitive disposition of the pupil. This disposition is not so strong when the pupil enters the high school as it was at any earlier age during his school life. Anyone knows that at the earlier age he is on the highroad in acquiring information of the things immediately about him. No subject is too profound for him to ask questions concerning. He can and does establish more connections with the external world in an hour, and understands them in his own way, than the adult either does or can in a much longer time. Both his mind and body are surcharged with a motive power that is asserting itself in many sorts of expressions. This energy expresses itself very differently in different children and to this difference must be given as much time and attention as possible if we, as we should, are

placing a premium on originality and individuality. Does not the teacher do much for the pupil and show his professional value when this inquisitive disposition has been retained by the child until he has reached us in the high school?

It sometimes seems that alertness of mind is lost almost in the inverse ratio to his school life. Certainly there is no more wholesome conception of the teaching process than to regard it as preserving this mental alertness, and it is in the preserving of this that *nature work* has its educational value. This alertness is inherent in the child's birthright and why should it be made less by being smoothed into a conventional type?

Second: A second value of this work is an unequalled opportunity to form a habit of close observation, and thereby obtain clear, distinct and accurate ideas, the only ideas worthy any effort. Such ideas can never be obtained by reading a book. From it one gets an idea of the writer's idea of the thing considered. As Mr. Huxley so happily expressed it when he compared the process of acquiring knowledge to vaccination, the virus used is purposely weakened by first passing it through the tissues of several successive animals until finally it is diluted enough to be robbed of its original violence. If passed through too many tissues it becomes so weakened as to lose all its preventative effects; so knowledge that does not come close from the original source becomes more and more uninteresting, meaningless and formal until it soon loses its vital power to keep the mind healthy and strong amid the many intellectual epidemics of the day.

Third: An excellent opportunity is offered in this work to make stronger and logical the natural tendency of concrete analysis. To anyone who has studied psychology the law is familiar that the mind, whether untrained or trained, perceives an object first, as a vague whole, then at once begins a concrete analysis. An example in which the mind may be directed and strengthened is in the study, say for the first time, of any type as the crayfish, in noting its position, size, shape, color, body segments, appendages and the number of each, and other points of detail analysis both external and internal.

Fourth: An excellent opportunity to make stronger the capacity for abstract thinking. When the boy has studied the anatomy of the grasshopper or butterfly, after having analyzed the whole into its related parts and thinking the part as separated from the whole, he is thinking in the abstract, a training

making more easy the thinking of abstract ideas in his mathematics later on.

Fifth: A fifth value is an excellent opportunity to make accurate and strong his power of discrimination. An ability to clearly and quickly discriminate so as to be able to express wherein a difference lies is rare and seldom possessed. This is a constant and trying feature of this work. The benefit derived is more than simply an ability to recognize a difference between a spotted cow and a butterfly in a field of clover. The nature and method of obtaining results in this particular phase of the work may be seen at a glance by referring to the table of differences.

Sixth: An excellent opportunity is offered to make strong an ability to see resemblances. So closely is this value related to the fifth that they might, with reason, be considered together; yet experience teaches me that pupils naturally recognize differences quite a great deal more readily than they do resemblances and for this reason they should be regarded as separately important. In fact it might be a fine point for psychology to settle and tell us whether we learn anything except by observing resemblances.

Seventh: The method of study used in this work involves a phase of training in which many particulars are incorporated into a general and comprehensive term; a training in the formation of general ideas. This is made obvious by referring to table two.

Eighth: The mind is working along the line in which is involved the method of forming logical conclusions and expressing them each in a clear, accurate definition. The importance of insisting upon accuracy in this particular phase of thinking may be noted on many occasions during the progress of a recitation. A large majority of pupils when asked any question which calls for an answer in the form of a definition, the following as an example, "What is an Isotherm?" will give this, "An Isotherm is *when* places having the same temperature are connected with a line." This defective definition is a type which reveals incorrect and illogical habits of thought. This work offers an unequalled opportunity to teach the pupil that a good definition has the following characteristics:

- (1) The definition must name the idea defined.
- (2) It must classify it by placing it into the smallest known class.

(3) It must state the essential characteristics which set it off from the other individuals of its own class.

It is readily seen how this work is adapted to this kind of training if you will refer to the table of information which the pupil should have worked out for himself before he can reach a logical conclusion and express it in the form of a definition. An example: An insect is an air-breathing arthropod having seventeen body segments which are divided into three body divisions, has a chitinous exoskeleton, one pair of antennae, two compound eyes, breathes by spiracles, three pair of legs, two of wings and moves by flying.

To sum up then the various values that may be made educational in teaching this work:—

(1) To leave the pupil with a trained ability to gather in for himself "first-hand" information.

(2) To inculcate the habit of obtaining clear, accurate ideas.

(3) To train him to think in the abstract.

(4) Concrete analysis, discrimination, comparison, generalization and recognizing logical identity.

Perhaps the instructor of english, of history or latin will be critical of the comprehensive value claimed for this work, but this in no way affects what may be accomplished. Certainly the many phases of training indicated should not be made exhaustive. The pupil, perhaps, should be kept unconscious of the fundamental purpose, or educational value, or end which the instructor should have in mind. It is his opportunity to so direct the work, to guide the pupil's interest, to control his energy, that he will go forth with a more extended knowledge of his own fitness and possibilities.

That it is the mental discipline involved and imparted and not the knowledge of grasshoppers and crayfishes, of beetles and bees which entitles this subject to so great an educational value, will be acknowledged by all. It is the training imparted and not the information acquired, just as the benefit derived from the study of a problem in geometry is not to be found in the information conveyed in the answer that he labors so diligently to obtain, and the value of the study of Latin comes not so much from the knowledge of the historical facts that the pupil learns while reading the language, so the value of the study of this work is not found in the knowledge of frogs and fishes, of birds and beetles, but in the systematically acquired mental ability obtained while gaining this knowledge, and unless this view of the

subject is kept in mind the true value and mission is being wrongly conceived.

To leave the discussion here is to fail to look far enough into the merits of the work to see wherein the benefits derived are carried into active life, enabling the pupil to more accurately and successfully adapt himself to the conditions and requirements in any field of life. When a pupil is required to talk and act upon what he sees to be true and to accept only what he finds to be true, we can see how this element of sureness alone strengthens the will. When he is allowed to spend his energy in the learning process without being required to keep constantly in mind the principle of cause and effect, there is great danger of his losing his orientation as to the forces, influencing his judgments and actions. Fancies and realities become confused, half the truth he will offer for the whole truth; this he reveals in his ambiguous expressions. This lack of clearness and accuracy vitiates his appreciation of the truth. Most strength results when reasons for one's actions are drawn from his own experience, and not from the fact that Washington or Lincoln or some other half mythical person would have done so and so under like conditions.

This method of training will build up a sense of thoughtfulness that the untruthful story of the cherry tree is powerless to effect. The boy is learning and acting from experience and is progressing in the very same way in which all his knowledge later on in life will have to be acquired. He learns what it means to stand alone and does not weaken as soon as he gets out of the schoolroom. He does not lose heart when the problems then present themselves, which no teacher can then illustrate for him with pencil and paper and names attached. He knows what it means and what it costs to get "first hand" the details which are so necessary and so is not so likely to become discouraged when unable to master so many new points in the old original way as he used to be obliged to when the lesson for each succeeding day was "six pages in advance." He learns how much care is needed, the close observation, what repeated verification and logical thinking are required to be unmistakably sure of the simplest fact and as a result he certainly is made less disposed to accept at face value the many pleasant exaggerations of the soulless salesman, the spell-binding politician or the confident real estate man.

One phase of the result of this work is exceedingly difficult to estimate. Formal examination cannot reach it as results may be reached in some other subjects. Throughout the work there is a slow and unconscious preparation made use of later in life in understanding the eternal fitness of things, in understanding nature in a larger way. An interest results leading to a desire to read with something more than mere entertainment such books as Kipling's "Jungle Books," Seton Thompson's "Wild Animals I Have Known," "Lives of the Hunted" and others of similar nature.

Finally no subject offers more opportunity to work with untouched originality and individuality than does this, and when pursued as indicated the tendency will be to send forth boys and girls in whose birthright is retained and strengthened these two genius making qualities; boys who are different from the conventional schoolboy type; boys who will not be mechanical duplicates of one another and who will not be so likely to be possessed with the weakening tendency to imitate but will seek chiefly to further unfold the possibilities of their own nature and follow fearlessly the calling of their own choice, reaching a more genuine enjoyment, a fuller achievement and a more ideal manhood.

Table of Differences

	ARTHROPODS		
	Insecta	Arachnida	Crustacea
Breathing.....	spiracles	spiracles	gills
Body segments.....	seventeen	eighteen	twenty
Body divisions.....	three	two	two
Exoskeleton.....	chitinous	hair	chitinous
Antennae.....	two	none	two
Eyes.....	sim.-comp.	simple	compound
Number of legs.....	six	eight	ten
Wings.....	two?	none	none
Locomotion.....	flying	walking	walking
Habitat.....	air	air	water

Table of Likenesses

ARTHROPODS	
Insects	{ Jointed appendages White blood Reproduce by eggs Jaws move sidewise Compound eyes
Arachnida	
Crustaceans	

**OPPORTUNITIES DUE THE SECONDARY SCHOOL TEACHER  
OF PHYSICS.\***

- (a). For Study and Improvement.  
(b). For the Preparation of Lectures and Laboratory Work.

BY DR. WILLIAM C. COLLAR,

*Roxbury Latin School, Roxbury, Mass.*

As I read the two topics, the thought occurred to me that your president has just expressed, that the two subjects did not seem to have a very close relation, they seemed, in fact, to be independent of each other; the first points to the advancement of the teacher, and the second seems to concern itself with the good of the pupils. But a second thought shows that there is an underlying idea which unites and binds these two topics together, that is, the topics mean opportunities for effective teaching of physics, or perhaps the most favorable opportunities for the most effective teaching of physics; and that formula may perhaps be reduced to simply this: Ideal conditions of teaching physics. And so I want to say a few words about the ideal conditions of teaching generally, and especially of teaching physics.

I am afraid you may wince a little at the word "ideal;" you may anticipate something that is too much in the air, something that has not its feet planted on the earth. I imagine you are thinking to yourselves, "Why talk about ideals? Our difficulties are of a practical sort, and what we want is a practical solution." In reply to that, I have to say that I think ideals are the most practical things of all—that ideals at least have their practical side. They ought to govern, guide, and inspire practice. I believe you all have ideals whether you realize them in consciousness or not; for without ideals, you would surely be like a man launching his bark upon the sea without a chart, rudder, or compass, and without knowing whither his voyage is tending. And if it were not for this practical side of ideals, I still say that we ought to cherish them with all fondness, as an antidote to the actual conditions in which we find ourselves. Our lives generally, our lives as teachers, and our lives specially as physics teachers, I imagine are often quite un-ideal, often depressing, if not discouraging, and we need to cherish ideals to forget, in some measure, the actual conditions under which we work. The

\*Address at the March meeting of the Eastern Association of Physics Teachers held at Roxbury, Mass.

cherishing of ideals tends to give courage and hope. I know it is so with me.

Now, the first of the ideal conditions that I wish to speak of is the condition of mastery of your subject. Do I know myself what I really mean by mastery of a subject? I can say one or two things that I do not mean, and that will bring us a little nearer. By mastery of a subject I do not mean knowing every possible thing about it, because I suppose however small might be the subject, to know all that is possible to be known would take more than a lifetime. Certainly the ramifications of every subject are infinite and countless in number, and I realize what our state as teachers is, and what is possible and what is not. Nor do I mean by mastery of a subject, that one should have the knowledge of an expert, who has devoted his life, perhaps, to some small part of human learning and teaching.

A teacher should know his subject accurately as far as he has to teach it and he must know more than he has to teach, or he cannot teach well, must know far more than he could expect to impart to any scholar. He must have a knowledge of his subject in perspective. He must be able to see the subject in its right relations.

What does mastery of a subject do for you? As a teacher, it must be of use in the presence of your class; it gives you confidence and that your pupils are sure to feel; it gives you power over yourself; it puts into your hands all the faculties that you possess. My own experience has been, that when I know my subject especially well, it induces a kind of elevation of spirit, and the effect of that is manifest in my class. I have not often to reprove my boys or to complain that they sit before me like graven images, irresponsive to my teachings. I do sometimes in my eagerness to know whether they really understand and appreciate, request them to smile or to wrinkle their brows, to make some sign that the thing is touching them. In general, that is not necessary; if I do feel in myself enthusiasm, it is infectious; the class feel it, and they show it in their responsive smiles.

Now, a second of the ideal conditions for the teacher of physics seems to be adequate time for his work and for study. I think that the feeling of the public, of school committees, and of many head masters is, that the more teachers teach, the better they earn their wage. This seems to be a grave mistake. I have acted on a very different principle, and have sought always to reduce the time my teachers had to teach, and

to limit as far as I could, the number of papers that they must correct.

I said last spring in New Haven, that in my judgment it ought to be agreed and understood that teachers are hired about as much to study as they are to teach, and I had the temerity to say I thought I was propounding a new doctrine, a strange gospel. But reflecting upon this I said to myself, "This is recognized and is actually practiced to some extent, now, in some communities, where they give the teacher occasionally periodical years to devote to study with half salary."

I remember several years ago, a professor in Harvard, who is eminent as a teacher and eminent as an editor, told me that he met classes ten times a week, and that it was altogether too much. I knew a good deal of his work, it was not unlike mine in classes in Homer. It was partly lecturing, but he said it was altogether too much, and he went to the corporation and said, "Gentlemen, I cannot do ten hours a week, it must be reduced to four, and if you find it needful to reduce my salary you may do so, but I cannot teach more than four hours." And I presume he was relieved of the ten classes a week, for he has been teaching four times a week for years.

Another ideal condition is, not too many pupils in a class or in a division. President Eliot, several years ago, is reported to have said publicly in Chicago, that he thought there should be not more than five pupils to a teacher. The thought seemed rather surprising, but when I reflected on the president's experience and observation, I was not so much surprised: as at that very time, there were only ten or twelve students to an instructor on the average in Harvard College proper, and in the Medical School there are three students to a teacher now. We cannot expect so few as that. I think it is, perhaps, desirable to have enough so as to have more or less of a spirit of rivalry in the class. Many years ago, Dr. Hale, twenty-three years a trustee of this school, came into my room and listened to a recitation, and he said, "It is all very pleasant and delightful, but you have too many pupils, you should not have more than fifteen." I had twenty-five pupils. I think Dr. Hale was pretty nearly right, and I should judge for a teacher of physics that that number is a fair average. And then a smaller number in a class or a division means fewer books to read, examine and mark. We must have tests and examinations, but to a large extent they are an evil, and not a thing desirable in themselves; and for a

teacher, after his day's work, or a part of his day's work, to examine many books or many papers, is certainly very wearying. Such work ought to be kept within reasonable limits, and that can be done if the numbers are not too large.

Another of the ideal conditions I should name is freedom. I think that fetters are not good for any human being, and so not for a teacher of physics. A man ought to be allowed, it seems to me, to find his own methods, to be free to produce his own results after his own notions. I presume that it may be true that if you are assistants, not heads of schools, you have liberty in the teaching of physics, because I imagine that head masters are too busy to look into the matter of methods, or if they are as ignorant as I am of the subject that they generally have the good sense to keep away and let the physics teacher do what he likes. Freedom you must have, and the most desirable freedom of all, is, perhaps, the exemption from bringing in all of a class or a division up to a certain point of knowledge on a given day. That is to my mind ridiculous. You have to struggle to do the impossible in a great many cases, and I think that the worst form of slavery. Now if any one should come into my Latin or Greek recitations, it might happen that he would find me interested in the argument of the oration of Cicero that we were on, or in the literary style, or what constituted the effectiveness of the appeal to the jury, or I might be wielding the syntactical rake and examining sub-junctives and ablatives, or I might be giving all my time to translation. "Why did you choose that word, what is the difference in force between this and that?" Meantime I should be seeming to exclude everything else, and so might a visitor say, and with apparent justice might make a very severe criticism. I have in that respect absolute freedom. I enjoy most of the ideal conditions that I am speaking of to you, and I am the only teacher in this school who does enjoy them. I teach fourteen out of thirty periods a week.

I will say in conclusion, that science has had to take a back seat in schools. I was in England sixteen years ago, and was interested in the subject of science teaching in the schools. I visited some of the great classical schools, and was surprised to find that science in some form was getting a place in them, but it was always in spite of the head master. I did not learn anywhere of a head master who showed any sympathy. They spoke of science with contempt, but I am persuaded that it is going to come to the front, and that very soon, and I say for your en-

couragement I believe that it is altogether an injustice that in any schools, especially in any classical schools, science should be depreciated. I want to read you a few words from an author whom, in general, I greatly admire:

"I hate and fear 'science' because of my conviction that for long to come if not forever, it will be the remorseless enemy of mankind. I see it destroying all simplicity and gentleness of life, all the beauty of the world; I see it restoring barbarism under a mask of civilization. I see it darkening men's minds and hardening their hearts."

I read that to show you how absolutely antipodal that is to my own feeling towards science. "It is and will be the remorseless enemy of mankind." Had the man not thought at all of the enormous alleviation of human suffering that we owe to science? I wonder if he forgot what we owe to science in the matter of sanitation and hygiene and the knowledge of how to live like human beings on this earth. "I see it darkening their minds and hardening their hearts." I think of a passage in Homer. In Homer the spirit of mischief, Ate, is personified. Ate strides through the world doing harm, and prays, Litae, follow after, correcting the evil that Ate has brought. As I think over the history of our race, I see how the dark and evil spirit of ignorance and superstition, like Ate, has borne heavily upon man through the ages, causing infinite woe; and then how in these days science like the Litae in Homer has followed after and is shedding, like a blessed angel, light and hope and comfort and peace upon mankind.

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A paper by Bruce V. Hill in the April *Physical Review* contains the following interesting statements:

"An alloy of 34 per cent nickel and 66 per cent steel forms the non-expansive alloy of Guillaume. Its coefficient of linear expansion is .000001. Block pendulums made of this material vary so little in length that a clock thus provided will run more accurately than other clocks with the ordinary compensating devices.

"The steels with 40 to 50 per cent of nickel have so nearly the same coefficient of expansion as glass that they may be employed when it is desired to seal a metal into glass. This material called 'Platinite' had already begun to replace platinum in the manufacture of incandescent lamps in France several years ago."

**SECONDARY SCHOOL GEOGRAPHY: WHAT SHALL IT BE?**

R. H. WHITBECK,

*State Model School, Trenton, N. J.*

The physical geography of the years prior to 1895 was fragmentary and unorganized. The now famous report of the Committee of Ten called for a radical reform. The schools endorsed that report. Text-book makers quickly adopted the new idea, and a new type of physical geography gradually came into use. This new physical geography was really physiography. It laid practically the whole emphasis on the classification and description of forms of land, upon meteorology and dynamic processes generally. It is very closely related to dynamic geology.

The most widely used of the early reformed text-books scarcely referred to man's relation to any of these land forms which were described. It was for the most part a very *earthly* science. Man was left out. The making of mountains, their denudation, the various kinds of mountains—all these matters were described, but the mighty influence of mountains in human affairs was not touched upon. This kind of geography does not seem to regard the earth as the home of man, but, rather, as an uninhabited globe, the seat of physical, but not of human forces. It is physiography, not geography.

In these same years the laboratory method in the teaching of science in secondary schools was rapidly gaining ground. Laboratory work was regarded as the right thing. Consequently, it was reasoned, the laboratory method ought to be applied to physical geography. I have a strong feeling that, if the exact truth could be known, we should find that much of the indoor laboratory work in physical geography was introduced, not so much because teachers actually felt the need of laboratory exercises to add clearness to the study, as because they felt that, somehow, a science because it is a science ought to be taught with laboratory experiments. Suitable exercises had to be devised. Laboratory manuals of all degrees of merit were published. I believe that it is the private, though perhaps not the publicly expressed opinion, of our experienced teachers of physical geography that the study is not lending itself readily to the laboratory idea. By the very nature of the subject, about the only laboratory is the field. Without a doubt there are profitable

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\*A paper read before the New Jersey State Science Teachers' Association, May 11, 1907.

laboratory exercises to be done indoors; the possible danger lies in bringing in kinds of work which are not profitable, all things considered.

It is quite the common practice, in schools where physical geography is seriously taught, to give one year of daily recitations to it. If this allotment of time be granted, then the question arises, What shall that year of study consist of? The following considerations will help in arriving at the answer: (1) That not one high school student in 5,000 will become an expert geographer and this one will go to college anyway. (2) Very few will become teachers of either elementary or secondary school geography without further study in a normal school or college. Moreover, not one person in 25 who enter the first year of the high school ever becomes a teacher at all. Manifestly the high school course in geography need not be shaped primarily to fit the needs of future teachers, though this may enter as a consideration. The fact is that the vast majority of those who study geography in the secondary schools, other than normal schools, are to become the men and women of affairs, salaried employees, home makers, professional or business men and women. It seems only reasonable, then, to decide that the course in geography ought to be planned with our chief thought for the interests of that great majority who are to be the men and women of domestic, social, civic, and business affairs. Considering the matter from this point of view, I would suggest the following: (1) That the *basis* of the high school course in geography should be physical geography with the elimination of many details now found in the text-books; this study of physical geography to be covered in about one half year. (2) That the second half year be devoted to the study of life relations, largely human relations, and involving a considerable commercial geography.

By a judicious sifting of what is really worth studying seriously, from that which is merely worth a thoughtful reading, any one of the good text-books in physical geography can be covered in a half year. If much of this is soon forgotten, as it will be, there is no harm done.

Text-books in physical geography, the things that largely govern our school courses, are undergoing beneficial changes. The changes ought to go on still farther in the same direction, as they unquestionably will. For example, one of the newest of the text-books gives 30 pages to a description of the fol-

lowing kinds of plains: Marine plains, lake plains, stream plains, waste plains, wave-cut plains, peneplains, and volcanic plains. It gives 28 pages to playa lakes, lava dam lakes, crater lakes, glacial lakes, delta basins and lakes, landslide lakes, coulee lakes, and lakes without outlets. To "The Importance of Lakes in Human Affairs," it gives about a half page. I cannot persuade myself that a detailed knowledge of all of these varieties of plains and lakes is at all necessary. Only specialists profess to know such details. The author of the book has adopted a wiser plan in his treatment of the subject of mountains. Here he has given half of the space to a chapter on "The Influences and Resources of Mountains." In this topic he has broken away from tradition, and has done so with undoubted gain.

The sweeping change in the character of Professor Tarr's *New Physical Geography* as compared with his earlier books is significant. His first and second books, written twelve or more years ago, contain practically nothing on the influences which physiographic and climatic factors exert upon the life and activities of man. The last book, written some five years ago, contains in nearly every chapter at least a brief discussion of human relations. He gives one long chapter to the topic, "Man and Nature," and another to the "Physiography of the United States," a thoroughly humanized chapter. Fairbanks, in his text book recently published, gives nearly half of the book to a discussion of the "Physiography of the United States," with constant reference to the life relations. This is a forward step, and one which is likely to be followed by others in the same direction.

What changes, then, seem logical and desirable with regard to secondary school geography? First, a pause; a candid consideration of the question, Is this study really a laboratory science to any extent? Is our present physical geography, with its details of seven kinds of plains, eight kinds of lakes, and nine kinds of mountains, sane geography? It may be, for those who are to specialize in physiography or who expect to teach it, but such students may justly expect the opportunity of studying it in higher schools. The secondary school course ought to be built on broader lines. I believe that the highest service that geography can do for the class of students whom we are considering, will be performed by making it a more broadly cultured study and less narrowly scientific.

Having had a half year of physical geography with a few laboratory exercises and much field work, if field work is at all possible, I should then like to have the course become quite largely a study of geographic influences: a few selected studies of the geographical control in historical events connected with our own country, to show the relation between history and geography. Such studies are found in Professor Brigham's "Geographical Influences in American History," or in greater detail in Miss Semple's "American History in its Geographic Conditions." I should like to have a study of valleys as the natural highways of exploration, of conquest, of travel, the places of fertile lands, the places where people choose to dwell, where cities grow up, where wealth will accumulate and great industries will thrive. A study of mountains as nature's great boundary lines of separation, and of isolation, the barriers to the migration of plants and animals and men, the last refuge of vanquished peoples, the shelter of the weaker tribes that flee to them and among them perhaps grow strong and free or, cut off from the world, live on for thousands of years, as have the Basques of the Pyrenees, preserving languages and customs which perished from the plains long, long ago.

A study of coast line and its influence upon the exploration of a new country, and its settlement; its influence through harbors upon the location of commercial cities, the gateways of a nation's trade, the points where the sea routes and land routes meet. A study of such specific cases as the fiorded coast of Norway and the evident influence of that coast in producing a race of mediæval vikings, and the best sailors of the world's modern navies. A study of the causes which lead to the locating of cities and their growth, to the localizing and the migration of certain industries. A study of the more evident effects of social, religious, and racial factors upon a nation's development of lack of it. Such influences are strikingly shown in the condition of China, India, Turkey, and Spain. Such studies in a secondary school are necessarily superficial, yet they are stimulating and broadening because they introduce great principles. They open one of the doors to culture. They expand the range of interests and widen the mental horizon. The study of the whole field of ancient history in the secondary school is superficial, yet it is exceedingly broadening to a young person.

**SOME EXERCISES IN LONGITUDE AND TIME.**

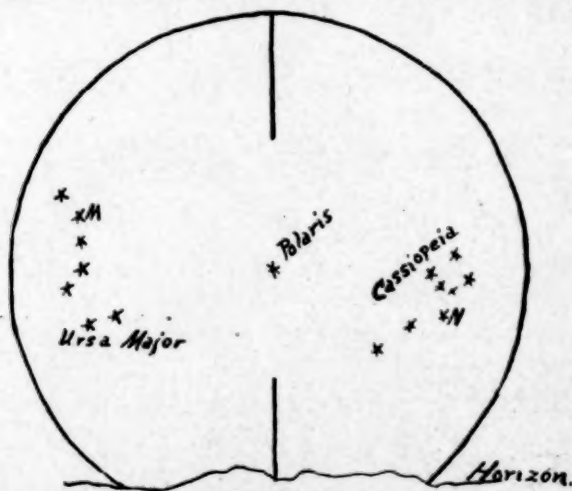
BY HERBERT BROWNELL, B.Sc.,

*Nebraska State Normal School, Peru, Neb.*

There are always some pupils in the high school in whom much text-book work has not stamped out originality of procedure and fertility in resource; pupils who, in spite of years spent in memorizing what the text-books say are the facts of their surroundings and what the book writers say these facts signify, still have the child's desire to find out some things for themselves and to think independently to some conclusions all their own. To such may safely be left the mastery of such exercises as the following with but little supervision by the teacher after the end in view and the conditions involved shall have been made clear. The enthusiasm of success in what has been undertaken and the inspiration to further effort resulting therefrom is catching among classmates. New fields of study are opened up and a fuller understanding is gained of what may perchance in words have been taught before.

## EXERCISE I.

*To Find the Longitude of Any Place.* 1. Out of doors in a place as much sheltered from the wind as possible and where a clear view of the heavens north and south is to be had, say on the east side of a building, suspend two plumb-lines two or three feet apart; let the weights that serve as bobs for these hang freely in pails of water to keep the lines steady. When the stars marked M and N in Fig. 1 shall be in vertical line with the Pole



Star as seen by looking past one of the plumb-lines, adjust the second line so that the three stars are in the common plane of the two lines. By sighting either north or south past the two lines it is easy to determine when any star is on meridian. These lines, if located in some suitable place, may be left there permanently for use at any time.

2. With watch set accurately for railroad (or "standard") time, observe and record to the second the times when first the west and then the east limbs of the sun come to meridian. The mean of these times *by the watch* is when the sun's center was on meridian. (Caution: Never look at the sun with unprotected eye. Use glass smoked or colored enough to prevent discomfort, and any danger to the eyesight.)

3. In some almanac find for that day how much the sun is "fast" or "slow" in minutes and (if possible) in seconds. If fast, add this "equation of time" to the time found by the watch when the sun's center was on meridian; if slow, subtract the equation of time. This sum (or difference) shows the time of the standard meridian when it is noon ("mean local noon") with the observer. The difference of time of the two meridians follows. Reduce this difference in time to degrees, minutes and seconds, and for places west of the standard meridian add it as longitude to such meridian; for places east subtract from the longitude of the standard meridian. Such result gives approximately the longitude of the observer.

4. Take the average of the results of at least three trials. From a good map determine as closely as may be the longitude of your station, and note the amount of your error.

#### EXERCISE II.

*To Correct One's TimePiece.* 1. From some good map find the longitude of the observer with all possible accuracy. From this the difference in longitude between the standard meridian and that of the observer follows, as does the difference in time.

2. Find as before (taking care to protect the eyes) the exact time by the watch when the sun's center is on meridian. To this time add the equation of time if the sun be fast, or subtract it if the sun be slow; to this result add the equivalent for the difference in longitude as found, if the observer be east of the standard meridian. (Subtract if located west.) This final result should be 12 o'clock exact, and any difference therefrom is the error of the watch and the amount of correction to be made.

3. Take the average of results on three successive days; then by comparison of your watch with railroad (standard) time note how nearly correct your average of results is.

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NOTE 1.—If these out-of-door plumb-lines be adjusted on the school grounds (say along the east or west sides of a school building), it will be easy to adjust *inside the schoolroom* at some south window a permanent meridian (a north and south plane). Suspend two plumb-lines from the window casing, one just outside and one inside, with bobs freely swinging in cans of water. With one observer at the plumb-lines out-of-doors to note the meridian passage of some selected star in the southern heavens, the plumb-lines within may be readily adjusted so that at the same instant the same star shall be on meridian for the observer inside. By repeated tests with different stars the lines within may be correctly adjusted and permanently fixed.

NOTE 2.—The true pole of the heavens through which our meridian passes is  $1^{\circ}.3$  from Polaris (the North Star) toward the star in the bend of the handle of the "big dipper" marked M in Fig. 1. As the single plumb-line extended upward marks the observer's zenith, and downward the earth's center, these two points and the position of the pole, when in a common plane, fix the observer's meridian.

NOTE 3.—The time of night when the adjustment of plumb-lines (as described) may be made varies with the time of year. These three stars are in vertical line in September about 1 A. M.; in November about 9 P. M.; in January about 5 A. M.; in March about 1 A. M.; in May about 9 P. M.

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With the plumb-lines at home the student may in the evening determine the latitude of his station; or, using those at the schoolhouse, he may reach like results by noonday observations.

**ESPY'S NEPHELOSCOPE.\***

BY CLEVELAND ABBE,

*Editor Monthly Weather Review.*

The experiment described by Dr. C. E. Peet in the April, 1907, issue of this journal implies the use of an air pump, whereas the following method, which has often been used by the editor, not only requires no expensive apparatus, but has several other advantages. A bottle (A), properly corked, has inside of it an ordinary elastic-rubber toy balloon (B), which, when but slightly distended, occupies only two or three cubic inches. A glass (or preferably a rubber) tube enters the mouth of the balloon, and also passes outward, air-tight, through the cork. On blowing through the tube, or forcing air by any other method into the balloon, the latter is distended, and of course the air within the bottle is compressed. Pinch the rubber tube and wait until this compressed air has lost its warmth, which it quickly does by conduction and radiation to the sides of the bottle, then remove the tube and allow the compressed air of the bottle to push the air within the balloon outward through the rubber tube. The work done by this expansion cools it enough to produce the most delicate cloud of condensed vapor, which is visible until the radiation of heat from the sides of the bottle evaporates the globules of water. The experiment may be repeated over and over with the same air always in the bottle; and if a thermometer be added, together with some way of measuring the volume of compressed air, then really instructive computations may be made. If a little water be kept in the bottle, but outside the balloon, we may so arrange as to deal always with saturated air, and the haze will be more easily visible to a large class. If no water be present then we have to deal with unsaturated air, and may make a large variety of experiments.

One of the first phenomena that the teacher and scholar will note is the fact that after a few trials it becomes more and more difficult to secure any visible haze. This is the phenomenon first recorded by Espy, and was a mystery to him and everyone else until Aitken showed that vapor condenses most easily on minute solid nuclei, and by its weight carries them to the bottom or sides of the jar, where they stick fast, so that after a few trials no more nuclei remain. Then comes the phenomenon first studied

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\*Reprinted from March number of *Monthly Weather Review*, page 123.

by C. T. R. Wilson of Cambridge, England, who showed that in dustless air a greater expansion and therefore a greater cooling is necessary in order to produce visible globules. This may lead us on to the consideration of ions, if the scholar is far enough advanced for the subject. At least it is proper to call his attention to the fact that the interior of a cloud is dustless, and that here greater expansion seems to be necessary, and consequently greater cooling, and that therefore a greater liberation of latent heat occurs within the interior of a thundercloud than in that same air when it first rises high enough to become cloudy.

Instead of water one may introduce other liquids into the experimental bottle, which is in fact a modification of Espy's single nepheloscope, and may thus experiment upon carbonic acid gas, the vapors of alcohol, ammonia, etc.

The double nepheloscope devised by Espy may be imitated by connecting two clear glass bottles (C) and (D) by means of two rubber tubes to a central bottle or receiver (E), from which the air can be exhausted. By a spring clip close one tube so that the air may be exhausted from the receiver (E) and one bottle (C), while not exhausted from the other bottle (D). Then remove the clip from (D) and allow its air to pass over into (E) and (C). The student will be surprised to find that no cloud is formed. This experiment troubled Professor Espy very much about 1850, as he had up to that time been reasoning on the general principle that the atmosphere is cooled by the act of expansion, but here he evidently had expansion without cooling. It was Prof. William Thomson, of the University of Glasgow, now Lord Kelvin, who, by his work on thermodynamics, first gave the true explanation, namely, that it is not the mere expansion that produces cooling but the work done by expansion. When the air expands from (D) into the vacuum (E) and (C) there is no work done except the moving of about one-half the mass of air in (D) over into the empty jars (E) and (C), and the cooling is too slight to produce a visible haze; it was, in fact, too slight for Espy to measure with his most delicate thermometer. On the other hand, when the compressed air in the bottle (A) pushes the air in the balloon (B) out into the open air it is doing heavy work by pushing against the outside atmospheric pressure, just as does the steam in the cylinder and boiler of an engine.

**DISCUSSION OF "COOLING BY EXPANSION."**

By H. E. HOWE,

*University of Missouri.*

The appearance in recent issues of this paper of notes on methods of showing cooling of gases by expansion makes it seem worth while to look a little farther into the principles underlying the phenomena described and those of cooling by expansion in general.

In the February number this experiment is described. A flask containing a little water is fitted with a cork rubber outlet tube, and pinchcock. The water is boiled, driving out most of the air, and the pinchcock is closed. There is now saturated vapor in the flask, slow boiling occurring as the vapor condenses. The rubber tube is connected to a second flask, the air is pumped out, and the pinchcock is opened between the flasks. Vigorous boiling occurs as a result of the decrease of pressure, and a fog appears above the water in the first flask.

What happens when the pinchcock is opened, and why does the fog appear? The pressure in the two flasks being very different, there will be a sudden passage of vapor over into the second flask. This vapor will move with considerable velocity, and each particle will possess kinetic energy equal to one-half times its mass times its velocity. This energy can only come from the vapor in the first flask, which, on account of this loss, will be cooled. Being already saturated, a part of it will condense to fog. The moving particles will stop their motion of translation when the pressure becomes equalized, their energy will all be transformed into heat, and a rise in temperature will occur in the second flask.

The vapor is *not* "cooled by expansion," taking that expression in its usual sense; that is, it has not changed temperature *as a whole* as a result of imparting energy to some other body. The experiment is nearly parallel with the historic experiment of Joule. Joule placed one cylinder containing gas under pressure in one water bath, and a similar cylinder, exhausted, in a second bath, and connected the two through a stopcock. The gas in the first cylinder was cooled; that in the second was heated. The statement in the article referred to that "the exhausted flask becomes filled with a dense fog" is evidently an error. In trying this experiment the fog formed was never

very dense, and was so obscured by the increased boiling that the experiment did not seem satisfactory for class-room demonstration.

In the April number Mr. Peet describes a similar experiment that shows fog formation better. A stoppered flask is placed under the receiver of an air pump, and the receiver is exhausted. The cork blows out, the air in the flask expands, and the water vapor present condenses. The fog shows very plainly, usually with only the normal moisture of the air, though when the humidity is very low a little water placed in the flask increases the effect. The cause of the cooling is the same here as before. The blown cork, moving with considerable velocity, will help make real to the student the conception of the kinetic energy of the issuing gas.

When a gas "cools by expansion" it must lose energy as a whole, *i. e.*, must do work on some other body. The simplest case of such an expansion is that of a gas enclosed in a cylinder and pushing a piston against pressure. If the pressure is  $P$ , the area of the piston  $A$ , and the gas in expansion moves the piston a distance  $x$  we have work = force  $\times$  distance =  $PA \times x = P$  times change in volume of the gas. The performance of this work requires the giving up of an equal amount of energy by the gas, which is thereby cooled.

An experiment more nearly approaching the above simple conditions was suggested to me by Mr. Peet's article. A strong flask or bottle is provided with a two-hole cork through which pass outlet tubes. One is closed with a pinchcock, and the other is attached to an ordinary bicycle or football pump. Air is compressed to about  $1\frac{1}{2}$  atmospheres in the flask, and the second outlet tube opened to the air. The air inside expands, and a dense fog appears. This fog lasts for about twenty seconds (much longer than in the other cases mentioned) and if the flask be placed in the path of light from a lantern or of a beam of sunlight, the fog can easily be seen from all parts of the room. The simplicity and success of this experiment should commend it to teachers whose supply of apparatus is limited.

**HIGH SCHOOL ALGEBRA.\***

HIRAM B. LOOMIS,

*Principal Hyde Park High School, Chicago.*

The main points I wish to make are that the work in mathematics for the first year of high school should center about the solution of algebraic problems, rather than upon the performance of abstract manipulations of symbols; that we should insist upon problems, problems, and then more problems, rather than upon manipulation, manipulation, and then more manipulation; that we should start with problems, find what manipulations are needed for their solution, introduce these manipulations as they are needed, and introduce nothing more during the first year: that we leave the rest of our present algebra, the abstract symbolism, for the latter part of the high school course. I should say that this juggling with symbols has no place before the subject of demonstrational geometry has received a great deal of attention.

At the present time, the first year of high school mathematics consists largely of mechanical manipulations. Milne's Algebra, the text used in the Chicago schools, has been examined with the following results: There are 41 problems on the first 5 pages of the book. From pages 13 to 135 there are 65 problems—one to every two pages. From pages 49 to 135 there are 27 problems, and from pages 67 to 134 there are only 15. Here we have nearly 70 pages of theory and drill work, with practically no problems. In other words, a few problems are given on the first few pages, as an excuse for the existence of algebra. I suppose; then we are expected to bow our heads to the fetish of symbolism. Nor is Milne alone in this. You can scarcely find an algebra in which there are not stretches of from 50 to 139 pages which contain not a single practical problem. The maximum number, 139, is from page 84 to page 223 of a high school algebra published since 1900; and the minimum of 50 pages is very conservative.

This is the condition so far as text books are concerned. This is the condition so far as our class rooms are concerned. The most abstract of abstract mathematics is placed at the beginning of the high school course; it belongs at the end.

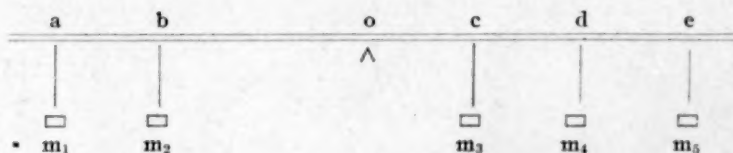
I shall not attempt to argue the pedagogical principle here

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\*Read before the mathematics section of the Central Association of Science and Mathematics Teacher, Nov., 1906.

involved. We are doing for the whole of algebra to-day exactly what one of the most recent texts does in its attempt to introduce problems in physics. As an example of putting the cart before the horse pedagogically, the following is hard to beat. A set of problems on moments is introduced in this way:

"When a straight bar is supported at some point, O, (Fig. 1) and masses  $m_1$ ,  $m_2$ , etc.,



are hung from the bar as indicated in the figure, it is found that when the bar is in equilibrium, the following equation always holds:

$$m_1 \cdot ao + m_2 \cdot bo = m_3 \cdot co + m_4 \cdot do + m_5 \cdot eo.$$

Then follow simple problems on teeter boards, etc. The most general equation possible first, and then simple numerical cases!

If we are to introduce the equation of physics in algebra, let us give the experimental basis first; let us follow that with numerous numerical problems, and gradually lead up to this general equation. Or, if it seems better, we may give all the algebra needed, and leave the physics to the physics teacher. Let me give an example of what can be done, one so simple that it can be given to pupils before they reach the high school.

5 apples at 2	cents apiece cost	$5 \times 2$	cents
3 " " 4	" " "	$3 \times 4$	"
10 " " $2\frac{1}{2}$	" " "	$10 \times 2\frac{1}{2}$	"
etc.	etc.	etc.	
leading up to			
10 " " p	" " "	10p	"
n " " 2	" " "	2n	"
n " " p	" " "	np	"

In this way we get the general equation  $np=c$ , in which  $n$ ,  $p$ , and  $c$  represent numbers. Then there are other series of simple examples leading up to the equations

$$n = \frac{c}{p} \text{ and } p = \frac{c}{n}$$

and there is an algebraic relation between these three equations. The area of a rectangle leads to a similar equation  $lw=a$ , in which, as before,  $l$ ,  $w$ , and  $a$  are always numbers.

Now let us turn to physics and notice the equations of this particular form.

$$v=gt \text{ [velocity = acceleration} \times \text{time.]}$$

(In every case the letter stands for a *number*.)

$$M=mv, \text{ [Momentum = mass} \times \text{velocity.]}$$

$$pv=c \text{ [Boyle's Law.]}$$

$$W=Fd \text{ [work = force} \times \text{distance.]}$$

I contend that if a pupil has the generalized arithmetic indicated above, he has all the algebra he needs for his physics; but abstract additions, subtractions, multiplications, and divisions, factoring, G. C. F.'s, and L. C. M.'s, the theory of exponents, radical equations, not to mention imaginaries, give no inkling that algebra is primarily a generalized arithmetic.

I have yet to find a pupil studying electricity, to whom the expression "difference of potential" means anything until a number of experimental cases have come under his observation. No amount of explanation will do the business; but in some way a little explanation coupled with considerable experience gradually leads to a grasp of the idea.

In algebra, pupils do not realize that the letters represent numbers. We may insist that pupils write "Let  $x$  = the number of units," instead of "Let  $x$  = distance;" but this will not answer; the pupil looks upon this as mere quibbling. I believe, on the other hand, that, if we have series of problems, beginning with numerical examples and ending with the most general expression possible in each case, the pupil will soon see that the letters in algebraic expressions represent numbers, because he has followed the real derivation of the formulae.

What would a first year algebra, centered about the problem in the way suggested above, look like? I shall leave you to answer this question; I simply hope that I shall make enough of an impression for you to notice, as problems come up in your algebra classes, how little and simple manipulation is required. Find a complicated case of addition, subtraction, multiplication, or division in the solution of a problem, if you can. Notice the

extent to which you need factoring. Keep a lookout for complicated cases of L. C. M's and G. C. F's. If you don't find problems giving sufficiently complicated manipulations to suit you, make up such a problem; but, if you have any regard for your pupils, don't try it on a first year high school class.

I hope before long to see an algebra based on this principle. I hope to see also a great variety of really practical problems, problems which the pupils will regard as practical; but, while this body of practical problems is gradually growing, we might as well use those we have now. I have heard teachers sneer at those old clock problems. Were it not for the fact that I should prefer a really practical problem, I can see no objection to the clock problems: A clock is certainly within the experience of the ordinary first year high school pupil, while the quantitative side of physical phenomena is not, unless the school itself gives a sufficient amount of laboratory experience. A great number of our practical problems will probably be derived from physics; but it is the height of folly to use them unless they are preceded by ample laboratory work. The most important part of the work is the derivation of the general formula from the laboratory experience. The substitution in this general formula is a matter of secondary importance. I grant that it is a necessary part of our work, of course; but a pupil, who has derived his own formula from his laboratory measurements, will have no difficulty in substituting for special cases. The point I wish to emphasize is this: In our desire for practical problems, let us be careful to keep within the real experience of the pupils. A paragraph on Boyle's Law in the algebra is not a sufficient introduction to justify algebraic problems which involve this law. A clock problem would be better.

Mathematics has been taught for centuries. We might think that the method of teaching would be nearly perfect by this time. The fact that only recently have instructors varied to any extent from the old Greek geometer, Euclid, is evidence that improvement has not been very rapid. All our mathematics, as we have it to-day, was put in shape for us by philosophers, by men who think in general terms. The history of Greek philosophy and the history of Greek mathematics deal with the same persons. And in modern times D'Alembert, Descartes, and Leibnitz are equally great in philosophy and mathematics. After these philosophers got their mathematics, they stated their conclusions in a logical order, not by any means the order in which they

reached these conclusions. We foolishly try to teach their generalization to first year high school pupils, on the theory that if the pupils once get these generalizations, they will be able to use them. The fact is that ninety-nine out of every hundred never think of applying the generalization when the time comes, and don't know how to do it when it is suggested to them. The contention of this paper is that the pupil should make his own generalizations; that he should make the generalizations *before* he applies them; and that only after he has had long experience in reaching general statements from a number of specific examples, should he think of taking any other person's generalization for granted.

If it is claimed that this is really a yielding to the demands of a practical age clamoring for a brand of mathematics that will "do things," I confess that it is; and I trust that the demands will be strong enough to force the exclusion from our elementary course in arithmetic, algebra, and geometry, of those sections of the subjects which do not succeed in "doing something" at the time they are introduced. And I express this desire not simply because I want the mechanic and artisan to receive what they need, but because I believe this will be the best possible training for the abstract mathematician. The material demands of a practical age are actually forcing better pedagogy in abstract mathematics.

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#### FREE TO EVERY TEACHER.

Few people realize what a valuable accomplishment it is to be able to use a dictionary with ease and certainty, so that in the hurry of daily life, whether in the school or in the home, it may be consulted without loss of time or studied with pleasure and profit in moments of leisure.

Most teachers fully recognize the value of the dictionary but how many regularly teach the use of the dictionary? The Publishers of Webster's International Dictionary have just issued a handsome, thirty-two page booklet on the use of the dictionary, "The Dictionary Habit." Sherwin Cody, well known as a writer and authority on English grammar and composition, is the author. The booklet contains seven lessons for systematically acquiring the dictionary habit. A copy will be sent gratis to any one who addresses the firm, G. & C. Merriam Company, Springfield, Mass. Should you not own a copy? Write to-day.

## CONSTRUCTION OF CONIC SECTIONS BY PAPER-FOLDING.

BY ALFRED J. LOTKA,

*Research Laboratory, Laurel Hill Works, L. I.*

A method has been described\* for constructing a parabola as the envelope of the creases formed on folding a piece of paper in such manner that a fixed point always falls upon a fixed straight line.

The other conic sections also can be similarly obtained, if for the straight line a circle is substituted\*\*, as is shown by the ac-

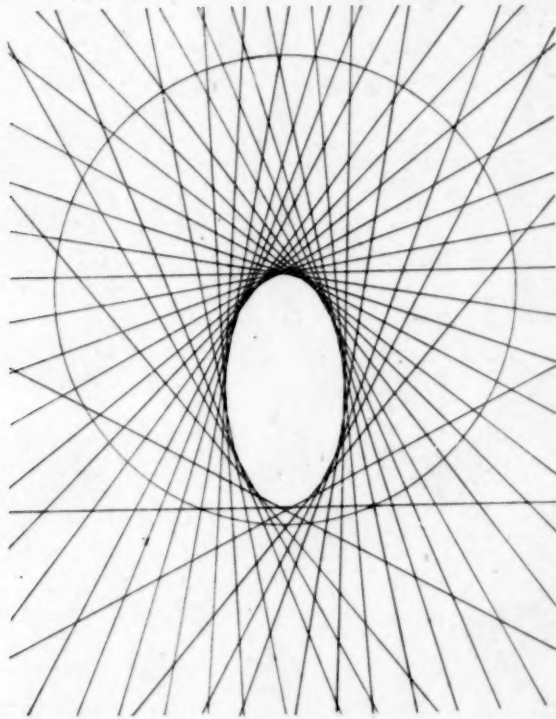


FIG. 1.

companying examples (Figs. 1 and 2), and by the following analytical demonstration.

Referring to Fig. 3, let  $C$  be the center of the fixed circle, and  $F$  the fixed point.

\*S. Row (W. W. Beman and D. E. Smith) *Geometric Exercises in Paper-Folding*, 1901, p. 116.

\*\*It is, of course, necessary to use translucent paper (tracing paper), or, if using opaque paper, to mark the fixed point on the back of the sheet, and on the edge of a perforation made in the same.

Bisect  $CP$  in  $O$ , and make  $O$  the origin of a system of rectangular co-ordinates, with  $OP$  for  $X$  axis. Let  $OP = x_0$ .

Then the paper is so folded that  $P$  falls upon some point  $Q$  of the circle.

If  $x_1y_1$  are the co-ordinates of  $Q$  we have:

$$(x_1 + x_0)^2 + y_1^2 = R^2 \dots \dots (1)$$

where  $R$  is the radius of the fixed circle.

If  $x_2y_2$  are the co-ordinates of the mid-point  $S$  of  $PQ$ , then

$$x_1 = 2x_2 - x_0$$

$$y_1 = 2y_2$$

Substituting these values in (1.) and simplifying:

$$x_2^2 + y_2^2 = \frac{R^2}{4} = r^2 \text{ say}$$

i. e. the point  $S$  lies upon a circle having its center at  $O$ , and a radius  $r = \frac{R}{2}$

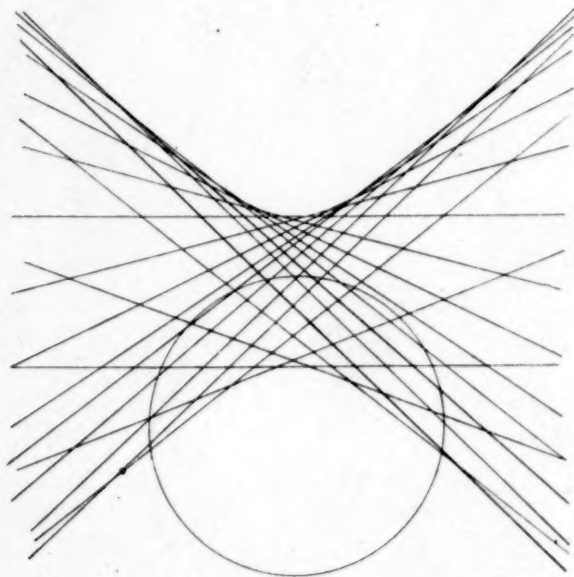


FIG.2.

Now the crease produced is evidently  $RT$ , perpendicular to  $PQ$  in  $S$ . Its equation is

$$y - y_2 = \frac{(x_0 - x_2)}{y_2} \cdot (x - x_2)$$

Rearranging and putting  $r^2$  for  $(x_0^2 + y_0^2)$ ,  $x_0(x + x_0) = r^2 + xx_0 - yy_0$ .

Squaring  $x_0^2(x + x_0)^2 = r^4 + 2r^2xx_0 + x^2x_0^2 - 2yy_0(r^2 + xx_0) + y^2y_0^2$

Putting  $x_0^2 = r^2 - y_0^2$ , simplifying, and arranging as a quadratic in  $y_0^2$

$$y_0^2 \left\{ y^2 + (x + x_0)^2 \right\} - 2yy_0(r^2 + xx_0) + r^2(r^2 - x^2 - x_0^2) + x^2x_0^2 = 0 \quad (2)$$

For a given pair of values of  $x$  and  $y$ , equation (2), gives either two imaginary, two real and different, or two real and equal roots for  $y_0^2$ .

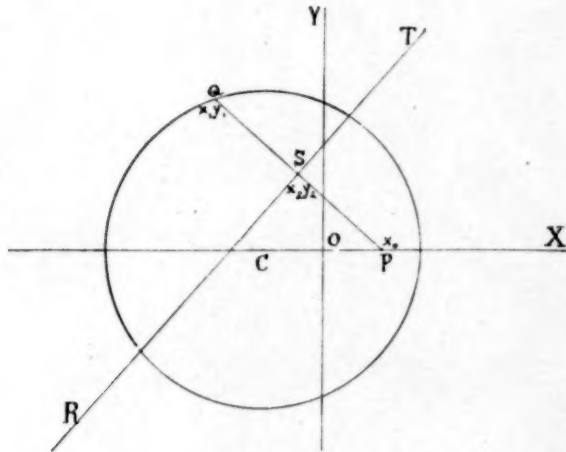


FIG. 3.

This means that the number of creases which can be produced through a given point in the manner specified is either 0, 2 or 1.

The points through which only one crease can be made evidently lie on the envelope of the series of creases, the single crease through each being tangent at such point to the envelope, so that the condition for this envelope is given by the condition for the equality of the roots of (2), viz:

$$4y^2(r^2 + xx_0)^2 - 4 \left\{ y^2 + (x + x_0)^2 \right\} \left\{ r^2(r^2 - x^2 - x_0^2) + x^2x_0^2 \right\} = 0$$

$$r^2y^2 + x^2(r^2 - x_0^2) - r^2(r^2 - x_0^2) = 0$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2 - x_0^2} = 1$$

Hence the envelope is an ellipse or a hyperbola, according as  $r > x_0$  or  $r < x_0$ , i. e., according as P lies within the fixed circle, or outside the same.

# THE SOLUTION OF $x=y=0$ OF THE EQUATION $\frac{a}{x} + \frac{b}{y} = c$ .

BY M. O. TRIPP,

*College of the City of New York.*

At the outset I wish to have it understood that I am dealing in this discussion with an algebra having a geometric correspondence in which infinity is included. Equations of the above type are usually placed under linear equations in our text-books, but it must be carefully noted that they are not linear in  $X$  and  $Y$  (see graph at the close of this article).

Putting our equation in the form

$$\frac{a}{x} = c - \frac{b}{y}$$

we see that as  $y$  approaches  $0$  the right side approaches  $\infty$  and hence  $x$  must approach  $0$ . Therefore we can write

$$\text{limit } x = 0$$

$$y = 0$$

Hence using limits we see that  $(0, 0)$  is a solution of our equation. This consideration answers the question of finding a value of  $x$  corresponding to  $y = 0$ . Its graph is a hyperbola passing through the origin.

This procedure is similar to the method of evaluating indeterminate expressions. Take, for example,

$$y = \frac{x^2 - 4}{x - 2}$$

then

$$\begin{aligned} \text{limit } y &= 4 \\ x &= 2 \end{aligned}$$

and hence we call  $(2, 4)$  a solution of our equation and consider that our curve passes through the point  $(2, 4)$ . On this topic see Granville\*.

There is good precedent for taking a solution as above. I cite an equation from the famous collection of Bardey†. He gives  $x = b$  as a solution of the equation

$$\frac{a-x}{x-b} = \frac{ab}{x^2-b^2} \quad (1)$$

This equation is equivalent to the two equations:

$$\frac{a-x}{x-b} = \frac{ab}{y-b^2} \quad (2) \text{ and } y=x^2 \quad (3)$$

\*Differential and Integral Calculus, Art 122 (1904). Ginn & Co.

†Algebraische Gleichungen Nebst den Resultaten u. den Methoden zu ihrer Auflösung. Fifth Edition, Leipzig, 1901. No. 204, p. 64.

From (2) we see that

$$\begin{aligned}\text{limit } x &= b \\ y &= b^2\end{aligned}$$

and the values  $x = b$ ,  $y = b^2$  satisfy (3). Hence  $x = b$  is a solution of (1).

In transformation by reciprocal radii, or in geometric inversion with respect to the origin as a center, a curve is made to pass through the origin in exactly the same way as our curve  $\frac{a}{x} + \frac{b}{y} = c$  passes through the origin.

Solutions like the above are closely related to the question of division by zero. With regard to this topic the reader is referred to the excellent treatise of Burkhardt\*\*. In this connection it is well to recall that  $(o, o)$  is a solution of  $ay + bx = cxy$  (4) only by agreement. I quote the following from Borel and Drach††: "We agree to write  $(b, o) = o$ . The product  $(b, o)$  has no meaning according to the definition given for the multiplication of integers." Fundamentally zero solutions are obtained from the idea of limits. In the case of equation (4) we may write

$$\begin{aligned}\text{limit } x &= o \\ y &= o\end{aligned}$$

From the preceding we get some results of especial interest in the teaching of Algebra. Take a system of simultaneous equations

$$\left. \begin{aligned}\frac{a}{x} + \frac{b}{y} &= c \\ \frac{a'}{x} + \frac{b'}{y} &= c'\end{aligned} \right\} \text{ I}$$

and consider its relation to

$$\left. \begin{aligned}ay + bx &= cxy \\ a'y + b'x &= c'xy\end{aligned} \right\} \text{ II}$$

1. The systems I and II are equivalent.
2. Clearing System I of fractions so as to get II does not introduce a solution  $(o, o)$  as is sometimes supposed. If we multiply  $F(x, y) = o$  by  $xy$  it does not necessarily follow, if  $F(x, y)$  is fractional, that thereby any one of the following solutions is introduced:

\*\*Funktionen theorie, 1903, Erster Band, Zweites Heft.

§ 11. Die Funktion  $\frac{1}{x}$  und die Transformation durch reziproke Radien.

§ 12. Die Division durch Null.

††La Theorie des Nombres et l'Algebre Superieure, p. 126.

$$x = 0, y = \text{constant}$$

$$x = \text{constant}, y = 0$$

$$x = 0, y = 0$$

for  $F(x, y)$  may become infinite under these values and hence the equation  $xy F(x, y) = 0$  is not necessarily satisfied, e. g.  $xy$

$$\left(\frac{a}{x} + \frac{b}{y} - c\right) = 0$$

cannot be assumed to represent the axes of reference and the locus of  $\frac{a}{x} + \frac{b}{y} - c = 0$ . We must distinguish carefully in our treatment between integral and fractional functions.

3. Usually system I is solved by considering it linear in  $\frac{1}{x}$  and  $\frac{1}{y}$ . This method of solution amounts to setting  $x' = \frac{1}{x}$ ,  $y' = \frac{1}{y}$ .

For every finite value of  $x$  or  $y$  there is a corresponding finite value of  $x'$  and  $y'$  except for  $x = 0$  and  $y = 0$ . The finite solution  $x = y = 0$  is carried by the transformation beyond finite limits and thus is lost in the infinite regions of the plane. This procedure is, I believe, very objectionable as a complete method of solving, since it causes the loss of a finite solution.

4. It is advisable to place equations like System I under quadratics where the student naturally looks for more than one solution and where by graphing he is better able to deal with the situation. In general the high school student can find two finite solutions of I. The other two are infinite owing to the parallelism of the asymptotes of the two hyperbolas. (See graph.)

It is sometimes supposed that it can be shown that the values  $(0, 0)$  do not satisfy an equation of the form we are considering. A careful investigation of all such statements will indicate, I believe, that there is nothing conclusive in them. As an example suppose we consider the system

$$\left. \begin{aligned} \frac{a}{x} + \frac{b}{y} &= c \\ x &= y \end{aligned} \right\}$$

From this system we get

$$\frac{a}{x} + \frac{b}{x} = c$$

but for  $x = 0$  this last equation is not necessarily equivalent to

$$\frac{a + b}{x} = c$$

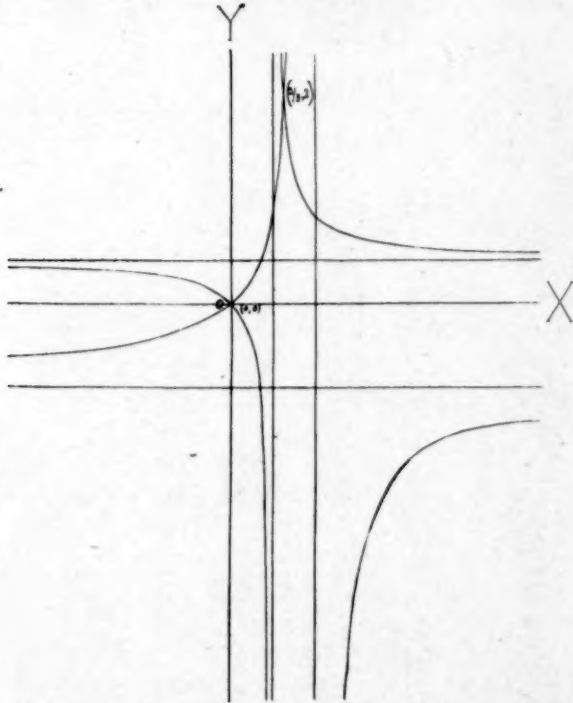
and hence from this it does not follow that the values  $(0, 0)$  do not satisfy our given system.

The following graph represents the equations

$$\frac{1}{x} + \frac{1}{y} = 2$$

$$\frac{1}{x} - \frac{1}{y} = 1$$

The solution  $(2/3, 2)$  is the one ordinarily given in our textbooks, while  $(0, 0)$  is not usually mentioned.



### AN IMPROVED ABACUS.

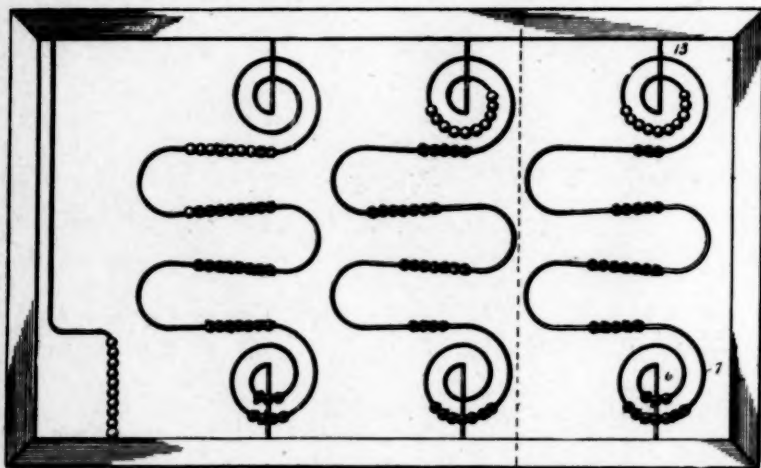
By STELLA E. MYERS,

719 Washington Ave., Kansas City, Kan.

The ancient abacus affords an intermediary process between full object work and the symbolism of Arabic numerals. In using objects our custom is to retain tens in groups; with the

abacus a ten is discarded and is symbolized by one in the next higher order.

In the abacus here illustrated, the patent for which is pending, balls colored in groups of ten are used upon wires and in the form of a series of compound curves. The right-hand wire is units, the next to the left is tens, and the third from the right is hundreds. The small wire on the left is for thousands in the result, these having been carried over from hundreds. A number is represented from left to right upon a series of horizontal curves, one on each wire. The small size of the abacus is designed for individual work and may rest upon the top of the school desk. On the large size, to be hung map fashion on the school-room wall, six numbers with orders up to hundred thousands may be indicated and a result reaching millions.



When an addition problem is represented with the balls the full groups of color upon units-wire are, in work with beginners, pushed upon the upper scroll; in more advanced work this is unnecessary. The part of a group remaining, if any, upon units-wire is pushed down to the lowest horizontal curve.

For every group of color or ten balls pushed upon the upper scroll of units-wire one ball is pushed from the lower scroll of tens-wire to the lowest horizontal curve of this wire. The process is continued in the same manner with the other wires and the whole result is read from the lowest horizontal curves of all the wires.

The children should do no counting of objects but should recognize at sight parts of the groups of ten and make combinations readily in two colors up to nine. Five is the largest part

necessary to recognize, as larger parts of ten are learned from the number, one, two, three, or four, that is missing from ten.

Before representing a problem in subtraction or division one group of ten beads is placed on the upper scroll of units-wire and another on the upper scroll of tens-wire to be used if necessary in place of one in the columns to the left.

The idea of multiplication may be given by using successive addends or, after the table is learned, by representing the products in each order and transferring tens. Object work in division is not feasible where more than one is to be transferred from one column to another.

This form of the abacus deals with the orders in the same relative position as in the written problem. Any other position confuses the concrete and the symbolical processes. As an illustration of the simplification of addition by this method, a child under four years of age that had been taught nothing of numbers was permitted to manipulate the beads according to the rules of the game for an average time of thirty minutes each day during one week. Then she could show the sum of six numbers in hundred-thousands in one half the time required by a fourth grade pupil working on paper. In the work on the frame there were no errors, while the written solution was not reliable.

There is naturally suggested by the use of this abacus a system of written addition which prevents mental fatigue in cases of continuous work.

2 3 6	2 3 6	In looking down units column the first num-
8 4 3	8 4 3	ber that helps with the preceding ones to form
6 5 7	6 5 7	a ten has drawn through it an oblique line.
7 2 5	7 2 5	With beginners or where an accountant is apt
3 1 8	3 1 8	to be interrupted the part of the number not
5 3 4	5 3 4	used in forming the ten is written to the right.
9 6 8	9 6 8	It is used in forming the second ten. The last
5 4 6	5 4 6	sum less than ten is written for the result in
4,827	4,827	the same column. The tens are marked out

of tens column in the same manner and the part of a ten remaining at the foot of the column together with a number of corresponding to the number of lines drawn in units column, the number of tens in that column, is written for the result in this column. The same method is then continued throughout the problem. Time is saved as well as energy. There is no occasion for inaccuracy, and consequently none for a test of the solution.

**A COMMUNICATION CONCERNING THE TYPE**  $px^2 + qx + r$ .

Essex High School, Essex, Ontario, May 27, '07.

Editors of SCHOOL SCIENCE AND MATHEMATICS,

440 Kenwood Terrace, Chicago.

Dear Sirs:

The question of precedence of the method of factoring the type  $px^2 + qx + r$ , as given in the April number of SCHOOL SCIENCE AND MATHEMATICS, has been raised by Prof. Smith in the June number. Whether or not this method was first published in the United States in 1900 as stated, it certainly was published some 14 years earlier in America. In "The Elements of Algebra," by J. A. McLellan, and published in Toronto, 1886, will be found on page 110, article 106, the identical method so well demonstrated by Mr. Toan in the April number.

The second method illustrated by Prof. Smith, or one very similar, is, I believe, quite generally taught to 3rd and 4th year High School students throughout Ontario.

The solution of the quadratic equation is taught about as follows:

$$\begin{aligned}
 px^2 + qx + r &= 0 \\
 \text{or } x^2 + \frac{q}{p}x + \frac{r}{p} &= 0 \\
 \text{or } \left(x + \frac{q}{2p}\right)^2 - \left(\frac{q^2 - 4pr}{4p^2}\right) &= 0 \\
 \text{or } \left(x + \frac{q}{2p} + \frac{\sqrt{q^2 - 4pr}}{2p}\right)\left(x + \frac{q}{2p} - \frac{\sqrt{q^2 - 4pr}}{2p}\right) &= 0 \\
 \text{or } x = -\frac{q}{2p} \pm \frac{\sqrt{q^2 - 4pr}}{2p}
 \end{aligned}$$

In the process of this solution the factors of the quadratic expression have been obtained, and when a formula becomes desirable the one will answer a dual purpose.

Thus to factor  $6x^2 + 11x - 10$  the student writes first the roots of  $6x^2 + 11x - 10 = 0$

$$\text{i. e. } x = -\frac{11}{12} + \frac{\sqrt{121 + 240}}{12} \text{ and } \therefore x + \frac{11}{12} - \frac{19}{12} = 0$$

$$\text{or } x = -\frac{11}{12} - \frac{\sqrt{121 + 240}}{12} \text{ and } \therefore x + \frac{11}{12} + \frac{19}{12} = 0$$

Hence factors required must be

$$6\left(x + \frac{11-19}{12}\right)\left(x + \frac{11+19}{12}\right) = 6\left(x - \frac{2}{3}\right)\left(x + \frac{5}{2}\right) = (3x - 2)(2x + 5)$$

Of course, multiplication by  $4p$  may be used instead of division by  $p$ ; but, having used both, I prefer the latter. Again referring to Mr. Toan's method of factoring  $12x^2 - 11x - 5$  compare this:

$$\begin{array}{ccc}
 12 & 6 & (4) \\
 1 & 2 & (3)
 \end{array}
 \qquad
 \begin{array}{cc}
 (5) & 1 \\
 1 & 5
 \end{array}$$

$$-15 + 4 = -11$$

$$\therefore 12x^2 - 11x - 5 = 4x \cdot 5(3x + 1)$$

The great majority of such examples are solved readily by the average student by trial. The above work indicates the method by trial reduced to a science. Write in order all the possible pairs of factors of the end coefficients repeating one set reversed i. e.  $\frac{1}{2}$  and  $\frac{1}{3}$ . Starting with  $\frac{1}{3}$  try cross products first with  $\frac{1}{2}$  then with  $\frac{1}{4}$  looking for a difference of 11. This failing  $\frac{1}{3}$  may be crossed out. Proceed in this manner until the coefficients of the factors are found. If every pair fails then the expression is not factorable.

Once more experience in teaching both Mr. Toan's method and this makes me use the latter almost exclusively.

Yours truly,  
R. W. ANGLIN, Prin.

### A COMMUNICATION.

Jersey City High School, Jersey City, N. J.

Prof. Ira M. DeLong, University of Colorado, Boulder, Colo.

My Dear Sir:—I was very much interested in the solution of the Frustum Theorem given by Dr. Blakslee in the December, 1906, number of SCHOOL SCIENCE AND MATHEMATICS, especially since I have for some time used a similar proof with my classes in Solid Geometry. I enclose my solution.

Yours very respectfully,

NELSON L. RORAY.

Lemma:—In a series of equal ratios the sum of the mean proportionals of each antecedent to its consequent is equal to the mean proportional of the sum of the antecedents to the sum of the consequents.

That is, if

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = \text{etc.}$$

then does

$$\sqrt{aa'} + \sqrt{bb'} + \sqrt{cc'} + \dots = \sqrt{(a+b+c+\dots)(a'+b'+c'+\dots)}$$

Proof:

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = \dots$$

$$\frac{aa'}{a'^2} = \frac{bb'}{b'^2} = \frac{cc'}{c'^2} = \dots$$

$$\frac{\sqrt{aa'}}{a'} = \frac{\sqrt{bb'}}{b'} = \frac{\sqrt{cc'}}{c'} = \dots$$

$$\frac{\sqrt{aa'} + \sqrt{bb'} + \sqrt{cc'} + \dots}{a' + b' + c' + \dots} = \frac{\sqrt{aa'}}{a'}$$

also

$$\frac{(a + b + c + \dots)(a' + b' + c' + \dots)}{(a' + b' + c' + \dots)^2} = \frac{a}{a'} = \frac{aa'}{(a')^2}$$

$$\frac{\sqrt{(a + b + c + \dots)(a' + b' + c' + \dots)}}{(a' + b' + c' + \dots)} = \frac{\sqrt{aa'}}{a'}$$

$$\frac{\sqrt{aa'} + \sqrt{bb'} + \sqrt{cc'} + \dots}{a' + b' + c' + \dots} = \frac{\sqrt{(a + b + c + \dots)(a' + b' + c' + \dots)}}{a' + b' + c' + \dots}$$

or

$$\sqrt{aa'} + \sqrt{bb'} + \sqrt{cc'} + \dots = \sqrt{(a + b + c + \dots)(a' + b' + c' + \dots)}$$

Q. E. D.

Let  $A'$  be the lower base of the frustum composed of the triangles  $a'$ ,  $b'$ ,  $c'$ .

Let  $A$  be the upper base of the frustum composed of the triangles  $a$ ,  $b$ ,  $c$ .

Let  $F$  be the frustum composed of the triangular parts  $F_1$ ,  $F_2$ ,  $F_3$ .

Let  $H$  be the altitude of the frustum.

Then

$$F_1 = \frac{H}{3}(a + a' + \sqrt{aa'})$$

$$F_2 = \frac{H}{3}(b + b' + \sqrt{bb'})$$

$$F_3 = \frac{H}{3}(c + c' + \sqrt{cc'})$$

$$\therefore F = \frac{H}{3}[(a + b + c + \dots) + (a' + b' + c' + \dots) + \sqrt{aa'} + \sqrt{bb'} + \sqrt{cc'} + \dots]$$

$$\text{Since } \frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = \dots$$

We have by the lemma:

$$F = \frac{H}{3}[A + A' + \sqrt{AA'}]$$

Q. E. D.

PROBLEM DEPARTMENT.

IRA M. DeLONG,

University of Colorado, Boulder, Colo.

Readers of the Magazine are invited to send solutions of the problems in this department and also to propose problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to Ira M. DeLong, Boulder, Colo.

ALGEBRA.

62. Proposed by W. T. Brewer, Quincy, Ill.

$$\begin{aligned} \text{Solve } x^{\frac{1}{2}}y^{\frac{1}{2}} + x^{\frac{4}{3}}y^{\frac{2}{3}} &= 132 \dots (1), \\ \frac{xy + 1}{x^{\frac{2}{3}}y^{\frac{1}{2}}} &= \frac{33}{8} \dots (2). \end{aligned}$$

I. Solution by J. Alexander Clarke, Philadelphia, Pa.

Dividing (1) by (2) gives  $\frac{xy + x^2y^{\frac{2}{3}}}{xy + 1} = 32$  whence  $xy=32$  or  $-1$ . From (2) it is apparent that  $xy = -1$  is an extraneous value. Factoring (1),  $x^{\frac{1}{2}}y^{\frac{1}{2}}(1+xy)=132$  and reducing by means of  $xy=32$  there results  $x=8$ ,  $y=4$ .

II. Solution by H. C. Whitaker, Philadelphia, Pa.

On account of the slight blurring in the print this problem may also be interpreted as follows:

$$x^{\frac{1}{2}}y^{\frac{1}{2}} + x^{\frac{4}{3}}y^{\frac{2}{3}} = 132, \quad xy + \frac{1}{x^{\frac{2}{3}}y^{\frac{1}{2}}} = \frac{33}{8}$$

Let  $x = z^3$  and  $y = v^2$ ; the given equations now become

$$z^4v^3 + zv - 132 = 0$$

$$8z^2v^2 - 33z^2v + 8 = 0$$

Multiply the first of these by  $8z$  and compare

$$v = \frac{1056z + 8}{41z^2}$$

Substituting in either equation

$$1177583616z^3 + 19440828z^2 + 216200z + 512 = 0$$

Solve this and substitute in the equation for  $v$

$$z_1 = -.0030503; v_1 = 12527$$

$$z_2 = -.0067297 + .0098615i$$

$$v_2 = -.16741 - 494.58i$$

$$z_3 = -.0067297 - .0098615i$$

$$v_3 = -.16741 + 494.58i$$

Writing  $\cos \theta + i \sin \theta$  as  $\text{cis } \theta$ ,\* these give

\*[ $c$  is the initial letter of cosine,  $i = \sqrt{-1}$  and  $s$  is the initial letter of sine. The abbreviation  $\text{cis } \theta$  for  $\cos \theta + i \sin \theta$  is used by Byerly in his Integral Calculus, and Stringham in his Uniplanor Algebra.—Ed.]

$$\begin{aligned}
 z_1 &= .0030503 \text{ cis } 180^\circ \\
 z_2 &= .012084 \text{ cis } 124^\circ 18' 37'' \\
 z_3 &= .012084 \text{ cis } 235^\circ 41' 23'' \\
 v_1 &= 12527 \text{ cis } 0^\circ \\
 v_2 &= 1745.6 \text{ cis } 196^\circ 27' 31'' \\
 v_3 &= 1745.6 \text{ cis } 163^\circ 32' 29'' \\
 x_1 = z_1^3 &= .000000028381 \text{ cis } 540^\circ \\
 x_2 = z_2^3 &= .00000176454 \text{ cis } 372^\circ 55' 51'' \\
 x_3 = z_3^3 &= .00000176454 \text{ cis } 707^\circ 4' 9'' \\
 y_1 = v_1^2 &= 156933000 \text{ cis } 0^\circ \\
 y_2 = v_2^2 &= 3047300 \text{ cis } 392^\circ 55' 2'' \\
 y_3 = v_3^2 &= 3047300 \text{ cis } 327^\circ 4' 58''
 \end{aligned}$$

63. *Proposed by I. L. Winckler, Cleveland, Ohio.*

A stone is dropped into a well, and after 3 seconds the sound of the splash is heard. Find the depth to the surface of the water, the velocity of sound being 1127 feet per second.

*Solution by Alfred Bjorkland, Lincoln, Ill.*

Let  $s$  equal the distance in feet to the surface of the water, and  $t$  the time in seconds it takes the stone to reach the surface of the water. Then solving for  $s$  in the equations:

$$s = \frac{1}{2}gt^2$$

$$s = 1127(3 - t)$$

We have,  $s$  equal to 133.67 feet, if we take  $g$  equal to 32.2.\*

### GEOMETRY.

64. *Proposed by L. K. Williams, Hannibal, Mo.*

Construct a triangle having given one angle, a side opposite to it, and the sum of the other two sides.

I. *Solution by T. M. Blakslee, Ph.D., Ames, Iowa.*

Let the given parts be  $A$  and  $a$ . On any line,  $CD$  take the segment  $CD = b+c$ , through  $D$  draw  $DF$  making angle  $CDF = \frac{A}{2}$ . In  $DF$  find  $B$  at distance  $a$  from  $C$ . In  $CD$  find  $A$  equidistant from  $D$  and  $B$ .  $ABC$  is the required triangle.†

II. *Solution by J. S. Brown, San Marcos, Texas.*

On the given side as a chord construct a segment of a circle capable of containing the given opposite angle. Also a segment capable of containing half the given angle.

From either extremity of this chord draw a chord in the larger segment above constructed equal to the sum of the other two sides. From the point where this chord crosses the arc bounding the first segment draw line to the other extremity of the given side.

65. *Proposed by I. E. Kline, Blairstown, N. J.*

Determine a point  $R$  within a triangle  $ABC$  such that angle  $RAB =$  angle  $RBC =$  angle  $RCA$ .

\*[It takes the stone 2.882 seconds to fall and the sound 0.118 seconds to rise.—E.D.]

†[If  $a < (b+c) \sin \frac{A}{2}$  there is no solution. If  $a = (b+c) \sin \frac{A}{2}$  there is one solution, if

$a > (b+c) \sin \frac{A}{2}$  there are two solutions.—E.D.]

*Solution by John P. Clark, A.B., Carthage, Mo.*

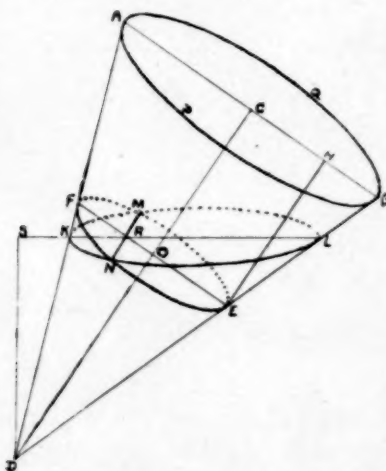
Draw the perpendicular bisector of AB. At B draw a line perpendicular to BC cutting this perpendicular bisector at P. With P as a center describe a circle with a radius PB. Draw the perpendicular bisector of BC and at C draw a line perpendicular to AC cutting this perpendicular bisector at Q. With Q as a center and QC as a radius describe a circle cutting the former circle at B and at another point R, the required point. In the circle Q, obviously angle RBC equals angle RCA and in the circle P angle RAB equals angle RBC. Therefore the required angles are equal.

### APPLIED MATHEMATICS.

66. *Proposed by H. E. Trefethen, Kent's Hill, Maine.*

A pail whose slant height is 10 inches, bottom diameter 8 inches and top diameter 13 inches is tilted to one side in a rain, so that the water caught rises from its lowest point 5 inches on the bottom and 8 inches on the lower side of the pail. Find the depth of water in the pail when set upright, and also the depth of the rainfall.

*Solution by E. L. Brown, M.A., Denver, Colo.*



Let FMEN be bottom of pail;  
LMRN surface of water.  $EB = 10$   
 $FE = 8$   $AB = 13$   $ER = 5$   
 $EL = 8$  In triangle HEB,  $EB = 10$ ,  $HB = 2.5$ ; therefore  $HBE = 75^\circ 31' 21''$  In triangle REL,  $EL = 8$ ,  $ER = 5$ ,  $LER = 104^\circ 28' 39''$ ; therefore  $RL = 10.44$ ,  $LRE = 47^\circ 53' 45.6''$ , and  $RLE = 27^\circ 37' 35.4''$ . In triangle KRF,  $FR = 3$ ,  $FRK = 47^\circ 53' 45.6''$ ,  $KFR = 75^\circ 31' 21''$ ; therefore  $KR = 3.48$  In circle FNEM,  $\overline{MR}^2 = FR \cdot RE = 15$   $\therefore MR = 3.87$   
 $KL = KR + RL = 13.92$   
 $ED : EO = EB : BH \therefore ED = 16$  and therefore  $OD = 15.49$   
In triangle LSD,  $DS = LD \sin DLS = 11.13$

Let  $a, b$  be the semi-major and semi-minor axes of ellipse KNLM. Then  $a = 6.96$ .

$$\text{Since } \frac{b^2}{a^2} = \frac{\overline{MR}^2}{KR \cdot RL}, \quad b = 4.47.$$

$$\begin{aligned} \text{Volume of water} &= \text{cone}(D-KMLN) - \text{cone}(D-EMRN) - \text{cone}(D-KMRN) \\ &= 362.78 - 170.64 - 70.86 \\ &= 121.28 \text{ cu. inches.} \end{aligned}$$

$$\begin{aligned} \text{Depth of rainfall} &= 121.28 \div \text{circle APBQ} \times \cos 56^\circ 34' 53.4'' \\ &= 121.28 \div 80 \end{aligned}$$

=1.36 inches.

Volume of cone D — FMEN = 259.54

Let  $x$  = depth of water in vessel when upright.

Then from similar cones we have

$$\frac{(x+15.49)^3}{(15.49)^3} = \frac{259.54+121.28}{259.54}$$

$$\therefore x = 2.11 \text{ ins.}$$

### CREDIT FOR SOLUTIONS RECEIVED.

Algebra 59. Mary L. Constable.

Algebra 62. H. E. Trefethen, I. L. Winckler, J. Alexander Clarke, Archibald Lindsey, E. L. Brown, W. T. Brewer (2 solutions), John P. Clark, Grant B. Grumbine, Orion M. Miller, Irwin E. Kline, T. M. Blakslee, H. C. Whittaker.

Algebra 63. H. C. Whittaker, E. L. Brown, Archibald Lindsey, C. E. Wheeler, H. E. Trefethen, I. L. Winckler, Grace MacLeod, T. M. Blakslee, John P. Clark, Grant B. Grumbine, J. W. Ellison, Geo. A. McFarland, Alfred Bjorkland.

Geometry 61. James A. Whitted.

Geometry 64. Alfred Bjorkland, J. F. West, Noah Knapp, P. R. Johnson, Grant B. Grumbine, E. L. Brown, H. E. Trefethen, J. Alexander Clarke, T. M. Blakslee, J. S. Brown, H. C. Whittaker (stone to fall: 2.882 secs.; sound to rise: 0.118 secs.).

Geometry 65. J. S. Brown, E. L. Brown, T. M. Blakslee, H. C. Whittaker, P. R. Johnson, Irwin E. Kline, H. E. Trefethen, I. L. Winckler, D. C. Colson, John P. Clark. One unsigned solution was received.

Applied Mathematics 66. E. L. Brown, H. E. Trefethen, H. C. Whittaker. Total number of solutions 53.

### PROBLEMS FOR SOLUTION.

#### ALGEBRA.

72. *Proposed by Wm. B. Borgers, A.B., Grand Rapids, Mich.*

At what time nearest after 6 o'clock are the hands of a watch in such positions that it would be possible for them to exchange places? And if they should exchange places, what time would it be then?

73. *Proposed by I. L. Winckler, Cleveland, Ohio.*

A man six feet high walks in a straight line at the rate of four miles an hour away from a street lamp, which is 10 feet high. Find the rate at which the end of his shadow travels and also the rate at which the end of his shadow separates from himself.

#### GEOMETRY.

74. *Proposed by H. E. Trefethen, Kent's Hill, Maine.*

Determine a point R such that lines drawn from it to the vertices of a triangle ABC make given angles with one another.

75. *Proposed by C. R. Merrifield, Grand Island, Nebraska.*

If a wheel has 35 cogs and the distance between the mid-points of the cogs is 12 inches, find the radius of the wheel. [From Phillips and Fisher's Geometry.]

#### TRIGONOMETRY.

76. *Proposed by J. M. Freeman, Boulder Creek, California.*

A point inside an equilateral triangle, side 50, subtends angles of  $140^\circ$ ,  $120^\circ$ ,  $100^\circ$  with the sides. Find the three lengths.

## DEPARTMENT OF SCIENCE QUESTIONS.

FRANKLIN T. JONES,

*University School, Cleveland, O.*

*This department is designed to serve as a medium for the exchange of ideas on questions and questioning in the sciences. Questions will be printed from various sources—college entrance examinations, text-books, etc., etc. Comment is invited. Suggestions and criticisms as to character, adaptability, and usefulness are desired. Readers of this journal are invited to propose questions and problems which will be of general interest, or of a type which will be useful in the class room. It is not expected that questions which will not be useful to pupils will be frequently printed.*

*Since the majority of the questions will be of a comparatively simple character, solutions and answers will not be published unless specifically asked for. Teaching suggestions are wanted.*

*Address all communications to the editor of the department.*

Teachers will confer a great favor on the editor by sending copies of the final examinations used in their schools last June.

In the November issue the June college entrance examination papers in chemistry will be published. The June physics papers follow. Teachers are invited to send in criticisms or commendations. For convenience the following outline is suggested:

- (a) Are the questions of the right kind? Are they tests of memory or of attainment? Are they too general? too technical?
- (b) Are the papers too difficult? too easy? Is the time allowed about right?
- (c) Do the questions call for a knowledge of principles, or merely for text-book knowledge? Are they on fundamentals?
- (d) Is a knowledge of more than can be taught in a single year of work required? What questions should not be asked?
- (e) Are the papers fair to both teacher and pupil?
- (f) Would physics teaching be better if all pupils had to look forward to such a test given at the end of the year by some examining body not the teacher?

## COLLEGE ENTRANCE EXAMINATION BOARD.

(Two hours)

In this examination 30 counts will be based on the laboratory note book submitted by the candidate and 70 counts on the following questions.

The candidate is to answer seven questions, as indicated below.

## A

*Answer two questions from this group.*

1. A string 200 cm. long<sup>\*</sup> is fastened at its ends upon supports 120 cm. apart, but at the same level; a weight of 4 kilos hangs upon the center of the string. What is the tension on the string? Make a diagram.
2. A bullet of mass 5 grams strikes with a velocity of 1,000 meters per second. How much energy is transformed? Define the unit in which it is expressed.

3. A box weighing 150 lbs. rests upon the floor of an elevator car: Would it exert a pressure of 150 lbs., or more or less than 150 lbs. on the floor—(a) When the elevator is rising with a uniform velocity? (b) When the elevator is descending with a uniform velocity? (c) When the elevator is rising with a uniform positive acceleration? (d) When the elevator is descending with a uniform positive acceleration? Give the reason for the answer in each case.

## B

*Answer one question from this group.*

4. 100 grams of steam at  $100^{\circ}\text{C}.$  are passed into 1 kilo of water at  $20^{\circ}\text{C}.$ ; what will be the final temperature of the mixture? (Latent heat of steam taken at 536.)

5. Explain with the aid of a diagram one of the devices which render the rate of a pendulum clock independent of changes of temperature.

## C

*Answer two questions from this group.*

6. If a pipe organ were tuned at sea-level and then transported to the top of Mt. Washington (6,000 ft.) and if the temperature remained constant, would it still be in tune? Give the reason.

7. How may two sounds differ in three particulars? How do the properties of the medium affect the velocity of propagation of sound?

8. Explain, using a diagram, why a straight stick appears bent when held obliquely with one end under water. Give another example of the same phenomenon.

9. An object is 50 cm. from a concave mirror. The image is 15 meters distant in front of the mirror. What is the relative size of image and object, and what is the radius of curvature of the mirror?

## D

*Answer two questions from this group.*

10. Explain two ways in which you could determine the electrical resistance of a coil of wire. State data to be observed in each case, and how the resistance is computed from the observed data.

11. What is meant by a "55 watt" incandescent lamp? If the lamp were on a 110 volt circuit, what current would it be using, and what would be its resistance under those working conditions? How many such lamps could be lighted by a 5 H. P. dynamo?

12. Make a diagram illustrating the construction of an induction coil, and outline its operation.

## CASE SCHOOL OF APPLIED SCIENCE.

(Ninety minutes)

1. Define force, work, energy, mass, weight, and give one or more examples or illustrations of each. What is meant by a 10 horse-power engine? an 8 horse-power boiler? a 25 kilo-watt dynamo?

2. A stone is thrown vertically to a height of 80 feet, with what velocity will it return to the level from which it started? Why is "g" less at the equator than at the poles?

3. What is the velocity of sound in the atmosphere? What effect does increase of temperature have on this velocity? Do sounds of different pitch travel in air with the same velocity? Describe "beats."

4. Define calorie, specific heat, latent heat. Explain why a great body of water is an equalizer of temperature. Describe a process for determining the latent heat of fusion of ice.

5. Describe the purpose of the lens in a camera. The focal length of a camera lens is 8 inches; a distinct image of a post is formed on the ground glass when it is at a distance of 10 inches from the lens: how far away is the post? Define conjugate foci.

6. Describe the principles involved in the production of a current of electricity in a simple dynamo. An incandescent lamp takes  $\frac{1}{2}$  ampere of current at 110 volts; what power does it consume?

HARVARD UNIVERSITY.

(One hour)

Omit four questions.

1. A hammer whose head weighs 500 grams and whose handle weighs 400 grams is submerged in water. The specific gravity of the steel head is 8. The specific gravity of the handle is .6. What is the force required to keep it from sinking or floating?

2. Why does frost occur on clear nights particularly?

3. Show by means of a diagram an arrangement of a lever whereby a man exerting a force of 50 lbs. upward could just lift a weight of 20 lbs. What other force acts on the lever? Show its direction and magnitude.

4. Explain the phenomenon of resonance and the action of a Helmholtz resonator.

5. If 1,000 grams of water at a temperature of 80°C. and 400 grams of mercury at 30° (specific heat 0.032) are poured into a vessel weighing 500 grams initially at a temperature of 20°C. and having a specific heat 0.1, what will be the temperature of the two, assuming no loss in the process?

6. What is the ratio between the cubical and the linear coefficients of expansion? Explain.

7. Define, with the aid of a diagram, the term index of refraction, and show what is meant by the statement that this index is always the same for a given color of light and a given material, for example, a particular kind of glass.

8. If the resistance of a wire 10 m. long and 1 mm. in diameter is .2 ohms, how great is the resistance of a wire of the same material 5 m. long and .4 mm. in diameter?

9. By a diagram show the connections of a Wheatstone's bridge. If the bridge is being used to measure the resistance of a coil  $x$  by balancing it against a resistance of 10 ohms and if balance is secured when the wire is divided by the bridge into two parts 72 cm. and 28 cm. right and left respectively as shown on your diagram, what is the resistance of  $x$ ?

10. Explain the method of artificial refrigeration or of making ice artificially.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

Time: two hours.

The numerical work as well as the answer is required in the solution of problems.

1. Explain meaning of the following terms: resultant force, moment of force, mass, power, parallelogram of forces.

2. A rod AB, 10 feet long, of uniform cross-section and weighing 10 pounds, is pivoted 3 feet from end A. What force must be applied at the other end to keep the bar horizontal when a weight of 20 pounds is hung at A?

3. What sort of motion exists in the following cases: (1) a body sliding down a frictionless inclined plane; (2) a body projected vertically upward in a vacuum; (3) an automobile in motion after the power of the engine is balanced by friction and air resistance? State reasons for your answers.

4. How much work is required to raise 500 gallons of water 100 feet vertically? One gallon of water weighs 8.3 pounds.

What horse-power would be required to do this work in 3 minutes?

Would it take more work to carry the water up an incline than to raise it vertically? Give reason for answer.

5. Explain a method for measuring the specific gravity of a solid and of a liquid.

How is it possible for a ship to carry a cargo of material that would sink if thrown overboard? What limits the weight of the cargo that can be carried?

6. Explain the construction and action of a mercurial thermometer.

If steam at  $212^{\circ}\text{F}$ . enters a radiator and after circulation leaves the same as water at  $212^{\circ}\text{F}$ ., will the room be warmed? State reasons.

7. Why does a pool of water seem to the eye less deep than it really is?

It is desired to project a real, magnified image on a screen. Would it be possible to use (1) a plane mirror, (2) a concave mirror, (3) a convex mirror? Give reasons for your answers.

8. State what will happen under the following conditions, giving reasons in both cases: (1) if a bar magnet is floated on a piece of cork and is free to move; (2) if a lump of iron similarly floated is brought near the above bar magnet.

9. Two incandescent lamps have resistances 120 and 240 ohms, respectively. What current will flow through each when they are joined (1) in series, (2) in parallel between two points maintained at a constant difference of potential of 120 volts?

PRINCETON UNIVERSITY.

NOTE.—Applicants presenting certified note-books will answer the questions under B only. Those not offering note-books must answer the questions under both A and B.

## A

1. State Newton's law of universal attraction.  
Why is the value of "g" less at the equator than at the poles?  
What is the relation connecting the weight of a body with its mass?
2. In the use of any machine, in what is there a saving, in work, in force, or in speed?  
What is the efficiency of a machine? the mechanical advantage?  
In the case of pulleys, how is the mechanical advantage related to the number of cords?
3. Explain the statement, "when ice is melted, heat is required and becomes latent."  
What is the heat equivalent of fusion? the heat equivalent of vaporization?
4. What is heat? State some of the reasons for your answer to the above question.  
Show how you can change the expression for a given temperature from the centigrade to the Fahrenheit scale.
5. Of what, essentially, does a dynamo consist? What determines the voltage it can give? What is the purpose of a commutator?  
State three ways in which an induced current can be set up.
6. What is reflection of light? What is the law of reflection?  
What is the principal focus of a lens? What are conjugate foci of a lens?

*Answer any five.*

## B

1. Resolve a force of 100 units into two components at a right angle to one another, one of which shall have a value of 50 units. What will be the value of the other?  
Show how to find the resultant of any five forces meeting at a point and making any angles with one another.
2. A body whose weight is 300 lbs. is raised in an elevator 70 ft. How much is its potential energy increased? In what units is your result expressed?  
What must be the horse-power of the motor which can raise the weight the 70 ft. in 5 sec.?
3. Describe the determination of the velocity of sound in air by the resonance tube method.  
A wire 96 cm. long makes 128 vibrations per second under a certain tension. What must the length become, if the number of vibrations is to become 512 per sec. while the tension remains constant?
4. Express  $72^{\circ}\text{C.}$  in  $^{\circ}\text{F.}$   $72^{\circ}\text{F.}$  in  $^{\circ}\text{C.}$   $27^{\circ}\text{C.}$  in absolute.  
70 grams of an oil at  $70^{\circ}\text{C.}$ , whose specific heat is .7, are poured into 400 grams of water at  $23^{\circ}$ . What will be the temperature of the mixture?
5. The volume of a certain amount of gas is 1,500 cu. cm. when its temperature is  $7^{\circ}\text{C.}$  If the pressure is kept constant, what will the volume become when the gas is heated to  $287^{\circ}\text{C.}$ ? When the gas is cooled to  $-133^{\circ}\text{C.}$ ?

6. The electrochemical equivalent of silver is .01118. How much of that metal will be deposited by a current of 1,000 amperes in 5 hours?

Change 1,000 kilowatts to watts; to horse-power; to ergs per second.

*Answer any five.*

#### SHEFFIELD SCIENTIFIC SCHOOL.

(Fifty-five minutes.)

1. Find the length of a lead rod having a mass of 1.52 kilograms, a diameter of 1.25 centimeters and a density of 11.3 grams per cubic centimeter.

2. A 15 gram bullet moving with a velocity of 600 meters per second penetrates 32 centimeters of wood. What is the average resistance (force) to penetration?

3. What is a barometer? Describe one or more of the familiar forms and their uses.

4. What is Boyle's law? What simple method may be employed for investigation of this law?

5. Describe the magnetic field due to a helix conveying a current. Show how to find the north pole of the solenoid.

6. What is meant by the fundamental and what by the overtones of a musical string? What influences do the overtones have upon the sound?

7. Describe and explain the colors of thin plates, or of Newton's rings.\*

#### ARTICLES IN CURRENT MAGAZINES.

Readers making use of this department will confer a favor on the editor by sending him a postal card to that effect.

*Farming* for June: "Brown Swiss Cattle—A Great Dairy Breed;" "What Has Been Accomplished by Milking Machines." For July: "Does a Small Flock of Sheep Really Pay?" "A Record-breaking Crop that Failed;" "Two Hundred Per Cent Profit in Forestry;" "A Working Plan for a Farm."

*Forestry and Irrigation* for May: "Forest Management in Europe;" "Colorado Short Course in Forestry;" "Inland Waterways Commission;" "The Norway Poplar;" "Work in a National Forest;" "Deforestation in Syria;" "Planting in California Forests;" "An Educational Tree Campaign;" "A Ranger's Cabin;" "Timber Tests by Forest Service." For June: "Forest Management in Europe;" "Appalachian-White Mountain Forests;" "A New Tree Juniper;" "Wood in the Vehicle Industry;" "The Jamestown Exposition;" "The Tree, the City, and the Citizen." For July: "Report of Women's Forestry Committee;" "Dead Timber in the National Forests;" "Convincing Testimony for Appalachian Reserves;" "Among the Shake-Makers in California."

*Garden Magazine* for August: "Crisp Celery from the Home Garden," photographs by Julian Burroughs, Nathan R. Graves and others; "Shall Evergreens be Planted in August?" photographs by W. McCollom, Nathan R. Graves and others; "Raising Perennials from Seed," photographs by Nathan R. Graves and Henry Troth; "Stonework without a Mason," photographs by the author and Claude H. Miller; "When Cannas are at their Best," photographs by the author, Henry Troth and E. J. Hall; "House-grown Daffodils for Christmas," photographs by the author and H. E. Angell; "A Timber Crop that Really Pays," photographs by the author; "The Simple Art of Budding Stone Fruits," photographs by Claude H. Miller.

\*Reprints of these questions can be had if ordered before October 1, 1907.

*Monthly Weather Review* for January: "The Climate of Yukon Territory;" "The Growth of Fog in Unsaturated Air." For February: "Meteorological Work at Camp Wellman, Danes Island, Spitzbergen." For March: "The Temperature in the Front and in the Rear of Anticyclones, Compared with the Temperature in the Center Area."

*Nature-Study Review* for April: "Nature-Study Development in Ontario." S. Silcox; "Foundation of Chemistry in Nature-Study," John Brittain; "Nature-Study as an Education," Mary P. Anderson.

*Photo-Era* for June: "Enlarging with the Lantern;" "Transparent Spots on Negatives, and How to Treat Them;" "Firelight Effects by Daylight;" "Combination Printing in Enlargements;" "A New Printing Paper."

*Popular Science Monthly* for June: "The Problem of Age, Growth, and Death," Professor Charles S. Minot; "The Progress of Our Knowledge of the Flora of North America," Professor Lucien Marcus Underwood; "Notes on the Development of Telephone Service," Fred LeLand; "The Value of Science," M. H. Poincaré; "Hygienic Requirements for the Printing of Books and Papers," Professor Edmund B. Huey; "The Waste of Children," Dr. G. B. Mangold; "A Blazing Beach," Professor D. P. Penhallow. For July: "What We Owe to Agassiz," Professor Burt G. Wilder; "Notes on the Development of Telephone Service," Fred LeLand; "The Great Japanese Volcano Aso," Robert Anderson; "Control of the Colorado River Regained," Charles Alma Byers; "The Value of Science," M. H. Poincaré; "The New Hygiene," Professor Wilfred H. Manwaring; "The Forms of Selection with Reference to their Application to Man," G. P. Watkins; "Illustrations of Mediaeval Earth-science," Dr. Charles R. Eastman; "The Progress of Science: Benjamin Franklin and the American Philosophical Society; The Celebration of the Bicentenary of the Birth of Linnæus by the New York Academy of Sciences; The State Universities and the System of Retiring Allowances of the Carnegie Foundation; Scientific Items."

*Review of Reviews* for July: "Rubber as a World Product;" "A Year of Delayed Harvests."

*School World* for June: "The Reform of Examinations;" "The Natural History of Animals;" "New Laboratories at the Whitgift Grammar School, Croyden, Eng."

*Science* for July 5: "Academic Freedom," by President Eliot. For July 19: "Linnæus and American Botany."

*Scientific American* for May 11: "The Manufacture of Matches in France." For May 18: "The Construction and Handling of Submarines;" "The Peril of the Broken Rail—Its Cause and Cure;" "Construction of Florida Coast Railway." For June 1: "Some Unknown American Natural Bridges;" "The Wonderful Sulphur Mines of Louisiana." For June 8: "Mining in Newfoundland;" "Nature's Touch-me-nots." For June 15: "The Transit of Mercury, November 14, 1907." For June 29: "The Heavens in July;" "Sound Signals for Mariners: An Inventive Field which is Not Overworked." For July 13: "How Linoleums and Oilcloths are Made."

*Scientific American Supplement* for June 1: "The Fate of the Temple of Philæ."

*Technical World* for June: "Smokeless Cities of the Future;" "New Colossus of Telescopes;" "Nature Fights the Railroads;" "Precious Stones at Home." Mrs. W. E. Burke; "Building a Lighthouse;" "Butter's Rival Gaining Favor;" "Explosives and Their Habits;" "The Opportunity the Small Farmer is Missing;" "Photographing the Human Voice;" "Latest Styles in Locomotives." For July: "Grim Guardians of our Coast," René Bache; "City Built on Rubies;" "Railroad Creeps out to Sea;" "Beautiful Caverns of Luray;" "Curbing a Hunan Flood;" "Fireworks in the Making;" "Marvels of High-Speed Steel;" "Government Ownership in Canada;" "Largest Hydraulic Gold Mine in the World;" "Farm on the Ocean Bottom." For August: "To Check the Gnawing Sea," Charles Frederick Carter; "Railroads Race to the North," Aubrey Fullerton; "Importing Feathered Songsters," René Bache; "Building a Butterfly Dam," William Hard; "Smelting Steel by Electricity," Henry Hale; "Fire, Axe and the Oregon Fir," Day Allen Willey; "Life-saving and Swimming Hints," Montague A. Holbein; "The World's Largest Bear," Lillian E. Zeh; "The Wizard of Fruits and Flowers," Louis J. Simpson; "When is Life Extinct?" Emmett Campbell Hall; "New City Built on a Jersey Marsh," Thomas D. Richter; "Awakening of the Chinese Giant," Owen Macdonald; "Old Feat Still Stirs Wonder," Philip S. Rush; "Making Cloth from Paper," Frank N. Bauskett; "Shall We Travel on One Rail?" William T. Walsh.

**REPORTS OF MEETINGS.****THE MISSOURI SOCIETY OF TEACHERS OF MATHEMATICS  
AND SCIENCE.**

The third annual (fifth regular) meeting of the Missouri Society of Teachers of Mathematics and Science was held at Columbia on May 4, 1907. A list of the officers for the coming year is given on another page of this journal. The annual dues were raised from \$1.50 to \$2.00. A resolution was adopted approving of the national organization of teachers of mathematics and science as far as already accomplished. A committee was provided for to report on the teaching of elementary algebra in secondary schools.

The following program was carried out:

**SCIENCE DIVISION.**

Paper: "Mushrooms." DR. B. M. DUGGAR, University of Missouri.

Paper: "Cathode Rays." DR. WILSON C. MORRIS, Warrensburg Normal School.

**MATHEMATICS DIVISION.**

Paper: "Beginning Algebra." CHARLES AMMERMAN, McKinley High School, St. Louis.

Discussion led by HERBERT P. STELLWAGEN, Yeatman High School, St. Louis.

Special Methods of Attack.

Paper: "Outlook for Mathematics." DR. E. R. HEDRICK, Missouri State University.

Paper: "The Equation in Geometry." A. A. DODD, Manual Training High School, Kansas City, Mo.

**JOINT SESSION.**

Papers: "The Articulation of the Teaching of Science and Mathematics in the Grades and in the High School."

(a) "The Articulation of Science," MR. S. A. DOUGLASS, Central High School, St. Louis.

(b) "The Articulation of Mathematics," DR. L. D. AMES, University of Missouri.

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**MEETING OF THE EASTERN ASSOCIATION OF PHYSICS  
TEACHERS.**

The Annual Meeting of this Association was held at Newton High School, Newtonville, Mass., March 23, being called to order by President Palmer with 41 members present. The secretary's printed report of the last meeting was accepted. He then presented the following annual report:

During the past year, the twelfth of the Association, two regular and three special meetings have been held.

The places of meeting were Simmons College, Massachusetts Institute of Technology, Everett High School, and Roxbury Latin School.

The speakers at the meeting of March 24, 1906, were Prof. Brown of Simmons College, who showed the Thordarson Electrical Apparatus

and other special pieces, and Prof. A. Wilmer Duff of Worcester Institute of Technology, who spoke on "Exposition, Experiment, and Discussion in the Teaching of Elementary Physics."

Mr. Gilley, at a special meeting May 19, 1906, addressed the Association on "The School for Instrument Makers at the University of Leyden."

The meeting of October 27, 1906, was a members' meeting, some twenty-two taking part, either in the review of a book or in the presentation of a piece of apparatus.

At the special meeting held at the Everett High School, February 16, 1907, the topic, "The Formal Education of the Future Teacher of Secondary School Physics," was discussed by Prof. C. A. Adams of Harvard University, Prof. H. M. Goodwin of the Massachusetts Institute of Technology, and Messrs. Black and Andrews.

On March 9, 1907, a special meeting was held at the Roxbury Latin School. The topic for discussion was "Opportunities Due the Secondary School Teacher of Physics." The speakers were Dr. William C. Collar of the Roxbury Latin School and Messrs. Charles S. Jackson, Manning, and J. W. Hutchins.

At nearly every meeting new apparatus has been presented by the Apparatus Committee and other members. The committees on Magazine Literature and Current Events have reported at each regular meeting. The committee on Reference Books has acceptably fulfilled its duty in the issuance of a separate pamphlet.

On February 2, 1907, there was a largely attended excursion to the Waltham Watch Works.

The lecture and laboratory course on Alternating Currents at the Massachusetts Institute of Technology has claimed the attention of many of our members. This course, like that of last year, was provided through the Lowell Institute.

Our membership has grown. In the death of Miss Girdwood the Association lost a valued member. Three active members have resigned and three have become associate members. One associate member has resigned. Six active and forty-eight associate members have been elected. There are at this time seventy-four active, sixty-nine associate, and two honorary members, a total of one hundred and forty-five.

The treasurer in his annual report showed the finances to be in a good condition with a balance of \$57.96 on hand.

The committees on New Apparatus, Current Events in Physics and Magazine Literature presented splendid reports showing that they had been active and their work helpful to the membership of the Association.

Mr. Cowen was continued as delegate to the Massachusetts Council of Education.

Mr. Coolidge, for the committee on the "New Movement among Physics Teachers," made a brief report, which was followed by some remarks on the same subject by Prof. Sabine.

Mr. Berry, explaining that SCHOOL SCIENCE AND MATHEMATICS had been obliged to raise the subscription price, moved that members

desiring it shall pay 50 cents in addition to the regular annual dues, if arrangements can be made with the publishers to furnish the magazine to a portion only of our membership at club rates. Carried.

Mr. Black was continued as associating Editor of SCHOOL SCIENCE AND MATHEMATICS.

The election of officers for the ensuing year resulted as follows:

President, Calvin H. Andrews, Worcester.

Vice-President, N. Henry Black, Roxbury.

Secretary, Frederick G. Jackson, Dorchester.

Treasurer, Percy S. Brayton, Medford.

Executive Committee, the foregoing, ex-officio; and Arthur H. Berry, Providence; John C. Packard, Brookline; Irving O. Palmer, Newton.

Professor W. C. Sabine of Harvard University then addressed the Association on: "Microscopic Vision and the Ultra-Violet Microscope."

At the afternoon session he gave notice that at the next meeting he would propose the following amendment to the constitution: The number of active members shall be unlimited.

Mr. Cowen was requested to act as a committee of one to use his influence in arranging a Lowell Course for Physics Teachers for next year along lines similar to those of the last two years, on the subject of Steam and Gas Engines.

A show of hands indicated that twenty-two of the members present would probably take the course on Engines, if it can be arranged.

Professor Francis C. Van Dyck of Rutgers College then addressed the Association on: "Physics Teachers Lectures."

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### ENCYCLOPEDIAS.

In this busy age no home should be without a good up-to-date Encyclopedia and Dictionary. The Chicago *Tribune* under date of June 14th said: "A good encyclopedia is no longer regarded as a luxury; it is a necessity in every home. As a storehouse of facts it is a valuable accessory to the family reading class as well as to the individual members of the household. It should be conveniently placed, and the children early trained to refer to it the moment they need it: information acquired under such a stimulus of interest usually is retained. It is a great mistake to make a list of subjects to be looked up some other time; a string of unrelated facts makes but a slight impression on the brain, and in addition to this objection is another—'some other time' is apt to be no time at all, and that particular list generally reaches the waste basket without being checked off."

We call the attention of our readers to a full page announcement of the United Editors' Encyclopedia and Dictionary in the present issue of this magazine. We own a set of this work and do not hesitate to recommend it to all those interested in this subject.

Complete information regarding it can be obtained by addressing a postal card to the United Editors' Association, 28 Jackson Boulevard, Chicago.

## BOOKS RECEIVED.

The Spirit of Nature Study, a book of social suggestion and sympathy for all who love or teach nature, by Edward F. Bigelow. Pp. 222. New York, A. S. Barnes & Co., 1907.

The Kansas University Science Bulletin, Vol. IV, Nos. 1, 2, 3, 4, 5, and 6. March, 1907. Lawrence, Kan.

Bulletin of the University of Texas, number 90, March, 1907. Austin, Texas. A Study in School Supervision with Special Reference to Rural School Conditions in Texas, by Carl Hartman, M.A., under the direction of N. S. Sutton, Professor of Education.

Throop Institute Bulletin, Pasadena, Cal., March, 1907. Reptiles of Los Angeles County, California, illustrated by Joseph Grinnell and Hilda Wood Grinnell.

Elementary Algebra, Part I. First year course. By C. E. Comstock, A.M., Bradley Polytechnic Institute, Peoria, Ill. 1907. Pp. 205. Published by the author.

The Moon in Modern Astronomy. A summary of twenty years' selenographic work, and a study of recent problems, by Phillip Fauth, with 66 illustrations and a frontispiece. Translated by Joseph McCabe, with an introduction by J. Ellard Gore. Pp. 160. London, 1907. A. Owen & Co., 28 Regent Street.

High School Physics (new edition), by Professor Henry S. Carhart, of the University of Michigan, and H. N. Chute, of the Ann Arbor High School. New edition, thoroughly revised. 12mo, cloth, 440 pages. Price, \$1.25.

## BOOK REVIEWS.

*High School Algebra. Elementary Course. By H. E. Slaught, Ph.D., Assistant Professor of Mathematics in the University of Chicago, and N. J. Lennes, M.S., Instructor in Mathematics in the Wendell Phillips High School, Chicago. Pp. xii + 297, Allyn and Bacon, 1907.*

This is the most pedagogical, the most practical, and the most teachable text for first-year high school pupils that we have yet seen. What may we not hope for for secondary algebra, now that two such eminent scholars of mathematics, both eager for the best possible university preparation in high school students, have broken with the traditionally classical point of view sufficiently to look at the subject from the view-point of the beginner! Here are two thoroughly competent men who believe there is a better way of enticing the mathematical snail from his tender shell than by jabbing him with a stick; that a circuitous route is more economical than a direct one, to refine the metaphor, to the storming of the heights of scientific algebra. These men have set a good pace for future text-book makers.

The chapter headings are: I. Introduction to the Equation; II. Positive and Negative Numbers; III. Involved Number Expressions; IV. Solution of Problems; V. Introduction to Simultaneous Equations; VI. Special Products and Factors; VII. Quotients and Square Roots; and VIII. Fractions and Literal Denominators.

The main purpose of this elementary course is declared in the preface to be "the solution of problems rather than the construction of a purely theoretical doctrine as an end in itself." An explicit attempt is made to relate the pupil's work in algebra to experiences of his daily life. Not all artificial problems are barred, however. It is recognized that some made-to-order exercises have an interest and value of their own.

Complicated factoring, and fractions, H. C. F. by successive division, simultaneous equations in more than 3 unknowns, fractional and negative exponents, and equations in complicated radicals are among the matters that are omitted from this treatment.

The room made by these omissions is taken up with practical problems from elementary science, and from modern affairs, with graphs, introduced at the proper place and made to do pedagogic service in simultaneous equations, and with more careful attention to the fundamentals. The authors seem to have a decided penchant for using the *Illustrative Problem* (in black-face type) to do the explanatory service that most text writers accomplish with more or less vague, usually more vague, general talk. Such vague talk, of course, only wearies a pupil. He reads it only if he is specifically required to do so, and he doesn't understand it when he does read it. This method of saying what one has to say to some specific point of subject-matter, rather than to some supposedly interested listener, is a great gain in efficiency.

The proportion of verbal problems to formal problems is much greater than is customary in elementary algebras. This is highly commendable. It is even a matter of concern with some whether there ought to be *any* of the formal, "unclothed," type of exercise at all. The authors, however, do not go to extremes. Enough of such exercises for the necessary practice are given; but they do not depend so completely upon them as is the wont, for the unfolding of the subject. They evolve the subject very largely through its thought-values, in contradistinction to its formal technique. There can be no doubt as to the soundness of this procedure.

Definitions are neither ignored, or overdone. Much induction is given. Still the authors seek gradually to cultivate deductive habits by giving the subject a founding upon some 18 ground principles. On the whole, the book is a great credit to the pedagogical insight of its authors. The writer does not believe the book has yet been excelled in point of pedagogic merit, of practical value, or of possible scientific fruitfulness. It deserves wide adoption. As to whether the plan of a separate book for the first and another for the third year is likely to succeed, only experience can tell. It augments cost a little, but the pedagogic reasons for it are sound and strong.

Allyn and Bacon have done a commendable piece of book-making in the general "get-up," typography (barring a few errors in the preliminary edition), and arrangement of matter on the page. The book is an all-around contribution to the pedagogical literature of secondary mathematics. We already hear of an unusually large number of adoptions.

M.

*Elementary Algebra. Part I. First-Year Course. By Clarence Elmer Comstock, A.M., Professor of Mathematics, Bradley Polytechnic Institute. Pp. 205. Clarence E. Comstock, Peoria, Illinois. 1907.*

In mode of presentation of topics, in the topics chosen for treatment, and in their order of treatment this book is out of the usual line of texts for beginners in algebra. The topics presented as well as the order of presenting them can be best judged from the following recital of chapter headings: I. Simple Equations; II. Quadratic Equations; III. Graphics; IV. Linear Equations Involving Two Unknowns; V. The Fundamental Operations; VI. Factors and Multiples; VII. Fractions; VIII. Roots, Exponents, and Radicals; IX. Logarithms; X. Imaginary Numbers; XI. Equations of One Variable; and XII. Equations of Two Variables. From this list of subjects it is plain that the author, who knows boys thoroughly, has given much more attention than writers of algebras for boys are wont to do, to the question of what topics a beginner can tackle with the greatest economy of time and effort and with the strongest feeling at the outset that "the game is worth the candle."

In a real sense this treatment not only accredits the validity of arithmetic in the algebra, but it actually "grows" the algebra out of the arithmetic. It will never enter the mind of the pupil, who gets his induction into algebra through this book, that the algebra is "something different" from the arithmetic. He will feel that he is expected at every step "to know all that he knows" up to the present moment. No whit of his arithmetic is "to be shelved" for a moment "to dry." Mensuration, valuation, and the fundamental operations of arithmetic are the points of departure of all topical treatments. The book as a whole is a homogeneous induction of the beginner into the field of algebra, tempered throughout to first-year possibilities. It is essentially a *first-year* book.

There is an abundance of good formal exercises, involving such a variety of symbols that there is small danger of the learner becoming enslaved to the tyranny of the traditional  $x$  and  $y$ . But, more noteworthy still, is the mode of approaching these formal exercises. There is nearly always treated one or more problems, clothed in a content of appreciable ideas, for which the equation is to be first formulated, and then solved. This procedure leads the learner to feel that even the formal exercises are the formulations of possible problems, and he never comes to look upon them as mere mechanical puzzles of no practical value. This concrete approach to lists of formal exercises might well be imitated by writers on elementary algebra. "First the *use* and afterwards the formal treatment" is the psychological order, though the common order is the reverse. Mr. Comstock's is the psychologic order.

The great care with which algebraic relations to be treated are first dealt with concretely through arithmetical numbers is highly commendable. The graph is early, extensively, and persistently used. The best of it all is that the graph is made to subserve teaching functions, not simply put in to pander to a poorly understood pedagogical demand

and then left. There are numerous problems from science properly organized into the algebra. The pupil is sure to know more arithmetic when he finishes this algebra than when he began it, and this is not a common virtue of algebraic texts.

Mechanically, the book leaves much to be desired. In later editions the book-maker will doubtless do fuller justice to the contents. The book deserves a wide audience among first-year mathematics teachers.

M.

*The Educational Significance of Sixteenth Century Arithmetic.* By Lambert Lincoln Jackson, Ph.D., Head of the Department of Mathematics, State Normal School, Brockport, N. Y. Pp. 232. Teachers' College, Columbia University, N. Y., 1905.

This book is the result of the study of the bearing of fifteenth and sixteenth century arithmetic upon the present teaching of the subject. It forms the eighth of a series of contributions to education published by Teachers' College of Columbia University. It consists of a valuable preface by the author, an excellent bibliography, alphabetically arranged, of the sources from which material for the research was drawn, a detailed table of contents and two very suggestive chapters. Chapter I is a discussion, filling 160 pages, on the "Essential Features of Sixteenth Century Arithmetic," and chapter II contains 40 pages on the "Educational Significance of Sixteenth Century Arithmetic."

The research covers the period from 1478, the date of the first printed arithmetic, to 1600, by which date arithmetic had assumed a stable form. No better period could have been chosen to correct a very common fallacy among schoolmen that school arithmetic is determined as to character of matter and mode of presentation from within by school officials and teachers. In this peculiarly formative period, it is clearly seen that the demands of out-of-school life, of commerce, trade, barter, industry, and the like, coerced it, oftentimes against the opposition of the school regimen, into the form in which it was most subservient to community needs. Once a new feature of arithmetic was forced into the schools by outside pressure, and the school succeeded in getting it organized, the teachers and school officers generally began to argue for it on the basis of disciplinary value. This so-called disciplinary value of study has always been an afterthought of teachers. Rarely indeed, if ever, does a subject first *get into* the school curriculum on grounds of its disciplinary value. The argument of disciplinary value is and has all along been merely a sort of didactic inertia exerted, often blindly, against any sort of change. The period during which this argument is most used, begins for any arithmetical topic just about the time when the community need that called for it and forced it into the curriculum, has passed away. This is not one of the author's points, but it is a lesson in pedagogics well substantiated by the historical evidence which he has gathered and used masterfully in pointing many another pedagogical moral, of which a modern teacher, who is solicitous of his professional standing cannot afford to keep himself ignorant.

If the time shall come when we are able to measure in some objective way just what the educational outcome of a good course in elementary school arithmetic really is, there will be much less vague talk of the *a priori* and hypothetical sort as to the virtues of such study. Until suitable standards for such measurement can be devised, the evidences of the history of arithmetic teaching are the most reliable means of guidance available to us. For this reason, Mr. Jackson deserves the unqualified praise of his colleagues for this study, and the Teachers' College is no less deserving for putting the results in such form as to be of the widest and most substantial service to teachers of the nation. Mr. Jackson had access to the best historical sources in this country for his study, and his findings are of the greatest practical value to modern arithmetic teaching. Every teacher of grade arithmetic should read this book. M.

*Space and Geometry.* By Dr. Ernst Mach, Emeritus Professor in the University of Vienna, translated from the German by Thomas J. McCormack, Principal of the Lasalle-Peru Township High School. Pp. 148. 1906. The Open Court Publishing Co., Chicago.

This little volume consists of three essays, written by Professor Mach for the *Monist*, and here made accessible to the public in unified form. They will be found extremely suggestive to teachers who are interested in the foundations of mathematics, or in the philosophical implications of non-Euclidean geometries. The titles of the three component parts are: I. On Physiological, as Distinguished from Geometrical, Space; II. On the Psychology and Natural Development of Geometry; III. Space and Geometry from the Point of View of Physical Inquiry.

These essays are in a high degree stimulating and clarifying to any who have gotten far enough along in their mathematical studies to have developed an interest in the essential nature of their science, and every thoughtful and growing teacher will want them on his book shelf. In this day of rapid progress, whatever conduces to clarity and deepens insight must be commended. The publishers have done praiseworthy service to mathematical pedagogy by putting these essays into book form. The 148 pages are packed with much of the subtlest thinking of the clearest mathematical heads and of the most powerful masters of abstract philosophy of all times, and modern teachers cannot afford to cut themselves off from their findings. M.

*Elements of Descriptive Geometry.* By Charles E. Ferris, Professor of Mechanical Engineering, University of Tennessee. Pp. vi + 127. American Book Company, 1904.

This is a very simple treatment of Descriptive Geometry primarily for technical students. It is concise, compact, and clear, and yet sufficiently comprehensive. The treatment conforms to the practice of many of the leading draftsmen, who do nearly all their work in the third quadrant. No doubt students may soon learn to think with work in this angle, and, once learned, it will be of permanent, practical value to them.

High school teachers should have such elementary practical texts as this on their shelves as sources from which to draw practical constructive exercises for their classes. Such exercises would be much more real, vital, and useful than the made-to-order type that accomplish little or nothing for the pupil when they are executed. Furthermore, such exercises are a far better training for the mathematical imagination than are the perfunctory type that fill so large a part of the time of the pupil of high school geometry. A live teacher could readily adapt exercises from this text to Euclidean purposes.

The publishers have done well their part of the work on the book.

M.

*Introduction to Metallurgical Chemistry for Technical Students.* By J. H. Stansbie, B.Sc. (Lond.), F.I.C., Lecturer in the Birmingham Municipal Technical School. Second edition, 1906. Pp. 252. Longmans, Green & Co., New York. \$1.25 net.

"This little book is intended for the use of technical students who are desirous of making a study of the metals employed for industrial purposes. It must be regarded strictly as a preparatory course."

The plan of the book assumes that those who use it have no knowledge of the common metals than that obtained by observation in a workshop or foundry. Only the more useful physical properties are mentioned.

The laboratory work begins with the effect of air and water on metals. This is followed by a discussion on properties of matter. Then follows in order the relation of sulphur and the metals, preparation and properties of sulphuric acid, the chlorine family, hydrochloric acid, nitric acid and the alkalies, relation between the common metals and acids, equivalents and atomic weights, oxides, acids, and salts.

Carbon and its compounds, reduction and combustion are each given a chapter. Phosphorus and silicon with their compounds are treated last.

The book is a combination of text and laboratory manual, with the theoretical principles developed as largely as possible from the experimental work.

The experiments are well written. The text is clear. Each chapter contains a well written summary with review questions.

A fine book to put into the hands of the bright boy who wants to do things. It should be in every high school chemical library. C. M. T. *Wellcome's Photographic Exposure Record and Diary*, 1907. Pp. 260.

Burroughs, Wellcome & Co., 45 Lafayette St., New York. 50 cents.

The book is a compact compendium of photographic information. It contains a pocket notebook, a diary, and ruled pages for systematically recording exposures, and a plate speed and light tables.

One of the most valuable features is an ingenious and practical exposure calculator by which the proper exposure can be instantly determined.

C. M. T.

*Rational Geometry. Second Edition. Thoroughly Revised. By George Bruce Halsted. viii + 273 pages. New York: John Wiley & Sons.*

Advocates of the study of geometry usually assert that its chief value is the cultivation of the power of reasoning, and that its conclusions are based solely upon rigorous logic. It is found upon examination that its claims are too often unfounded, and that many propositions are simply abandoned with a Q. E. D. A book treating of elementary geometry from the standpoint of modern developments of the subject is a most welcome and valuable contribution. Such is the unique position of the *Rational Geometry*.

To teachers who imagine their work is rigorous, or who may desire to investigate methods by which current demonstrations can be improved, this book is indispensable; it should be in the hands of every teacher of elementary geometry. In it we find not only the well-known theories that a straight line cannot intersect a circle in more than two points, but also the equally important theorem that a straight line cannot intersect a triangle in more than two points; as well as many other theorems which are necessary for a rigorous treatment of the subject but which are generally entirely overlooked and unsuspected.

The chapter on constructions should prove of immediate interest. In it are constructions without the use of a circle for nearly all the problems of elementary geometry; the bisection of an angle and the construction of a parallel to a given line are especially neat.

If the work is at present too deep and exhaustive for beginners, we must remember that a few centuries ago to perform ordinary long division was considered quite difficult, while one book of Euclid was about the maximum attainment of those who qualified for the master's degree. No one can help but find his teaching of any proposition illuminated by a study of its treatment in this text.

G. W. GREENWOOD.

*The Teaching of Mathematics, by J. W. A. Young, Ph.D., Assistant Professor of the Pedagogy of Mathematics in the University of Chicago. Longmans, Green & Co. Price, \$1.50.*

For the teacher with or without experience this is, perhaps, the most helpful book on its subject in English. The treatment, while not exhaustive, is adequate; particular methods are considered in the light of general principles, and their description is detailed and specific. The tone is temperate and judicial; the whole discussion is stimulating and suggestive; just the book to commend to a young teacher; also to a more experienced one who is just settling comfortably into the ruts. It is the work not of a doctrinaire but of a skilled and practical teacher who is at the same time a mathematician of high rank.

The discussion opens on the need of pedagogic training on the part of prospective teachers, and passes to a consideration of the value of the study of mathematics. This chapter is admirably adapted to remove misapprehensions on the part of educators as to the nature and value of mathematical training, misapprehensions which forms the

staple of much of the current discussion of mathematics as a part of school and college curricula.

Coming to the more immediate object of the book, we find more than one-fourth of it devoted to "Methods and Modes." By "method" is meant "the manner in which the subject matter is arranged and developed"; by "mode," "the manner in which it is presented to the pupils." On mode the author has this observation: "*If the mode used is such that the pupil makes no more progress than he would have made without the teacher, this on the face of matters condemns that mode under those circumstances.*"

The heuristic method receives careful treatment. As to its value the author remarks: "Of the considerable number of teachers who have to my personal knowledge used the heuristic method, none have abandoned its spirit. They have modified detail, as guided by experience, but only in order more effectively to awaken and cultivate the heuristic spirit in themselves and in their pupils. Numerous teachers testify that the weaker pupils profit specially by this method."

To many the "Laboratory Method" connotes something hazy and undefined, a recent and, perhaps, deplorable innovation in the methods of mathematical teaching. All such will turn with interest to the chapter on the "Laboratory Method." The careful and conservative attitude of the author is manifest. The method is given a fair and full statement, impartial and impersonal, though I suspect that the chapter affords the best example of the author's own constructive work to be found in the book. The keynote of this method is *interest*; the fundamental principle, closer relations among the several mathematical subjects themselves and between these and their applications to the physical sciences. "The laboratory method goes a step further and makes the most radical proposition so far made anywhere, viz., that mathematics and physics be organized into one coherent whole, the most extreme form of the proposition being that the reorganization be so thorough as to recognize in the secondary school no distinction between mathematics and its principal applications. The laboratory method proposes that the experimental origin of mathematics be fully recognized; that the pupil be led to feel the need of the mathematical tool through some material experiments he has made or things he has done. He thus abstracts his own mathematics. The power of abstraction so developed, very simply and gradually at first, must be used more and more freely. The laboratory method while insisting on the experimental, the concrete, the workshop side of mathematics, and while at present laying *special* emphasis on these things because they have been too long under-emphasized or entirely ignored, is no foe to abstract mathematics and does not aim at eliminating any abstract mathematics simply as such." One finds here also an account of the mathematical laboratory, examples of graphical work, drawn largely from Moore, Cross-section Paper as a Mathematical Instrument, a rich storehouse of such examples, together with many other practical suggestions for carrying the method into operation. It is pointed out that this method is in line with the trend of recent mathematical

pedagogy in England, France, and, particularly, in Germany, where the influence of Klein is strongly felt.

The teaching of mathematics in a considerable part of the country has been greatly influenced by this method in the last few years, and this influence bids fair to increase rapidly. Whether the method as a whole be adopted or not, every teacher and every school will find in this chapter much that can be used. The general adoption of the method in its extreme form, involving, as it does, the equipment of a laboratory and specially trained teachers, will be necessarily slow. However, the deep interest in the subject fostered by the many organizations of teachers of mathematics in this country, will make the trial of the method easier and more rapid. And, perhaps, only after a trial on a somewhat extensive scale has been made under the conditions prevailing in this country, can a just estimate be given of the ultimate form the method will assume. In the opinion of the writer, the wide separation of mathematics from its applications, as commonly taught, is in considerable measure responsible for the outcry against mathematics in high school and college. And unless mathematics is to take its place alongside Greek in popular estimation, teachers must see to it that elementary courses in pure mathematics are kilt closely into their applications in the physical sciences. This will also tend to reduce the distressing proportion of students who know mathematics memoriter only. So far, also, as the laboratory method serves to bring mathematics into true relations to the physical sciences, it advances the interests of both mathematics and these sciences.

Other chapters are: Preparation of Teachers, The Material Equipment, The Curriculum in Mathematics, Definitions and Axioms, The Teaching of Arithmetic, The Teaching of Geometry, The Teaching of Algebra, Limits.

Space does not permit a detailed account of these chapters. They form a thoughtful discussion of questions of great importance to teachers, and will repay careful study. There are many valuable suggestions and hints in the chapters on the teaching of Arithmetic, Geometry, and Algebra. The last chapter on the difficult subject of the teaching of the theory of limits in elementary mathematics, will be found stimulating and helpful.

The value of the book is greatly increased by the bibliographies which head the chapters and by numerous notes giving further references or fuller quotations. There is, also, a good index, which, together with the full table of contents makes it easy to find any desired topic. The book contains approximately 350 pages, and is well printed. It belongs to the American Teachers Series, edited by Dean Russell, Teachers College, Columbia University; maintains the high standard of excellence set by the other volumes of the series; responds to a widely felt need for information on the topics discussed; and ought to be in the library of every teacher of mathematics and every superintendent of instruction.

T. E. M.

*Questions and Problems in Chemistry compiled from college entrance examinations with a list of incomplete equations, by Franklin Turner Jones, University School, Cleveland, Ohio, 1907. Pp. 28.*

Wholesale price, 30 cents.

This little book is similar to the Questions and Problems on Physics by the same author reviewed in our June number. It contains 312 questions and problems compiled from the various college entrance examining boards.

Part II contains 280 incomplete equations. The book fills a long felt want in reviewing for final or college entrance examinations.

In my own classes I have found it to be a very great help in arousing interest and enthusiasm in review work. Every chemistry teacher should not only possess a copy but also insist that each pupil has one.

C. M. T.

*Practical Zoology. Alvin Davison. American Book Co. 1906. 368 pages. 396 figures.*

This is a text-book intended for secondary schools and for use in connection with courses extending through either one half year or a whole year. An introductory section on equipment and classification is followed by about a hundred pages devoted to the treatment of insects chiefly from the standpoints of their economic relations to man and their more striking features of structure or habit. The little laboratory work outlined is on the grasshopper and consists chiefly of statements concerning the terminology and certain more obvious facts of anatomy which the pupil may verify by observations on his specimen or by a study of the figures in the book, if specimens are not provided. Other groups of invertebrates are then taken up in much the same way, proceeding from higher to lower forms including Protozoa. About one hundred pages are allotted to the vertebrata, and sixty pages to the discussion of such topics as development, senses of animals, protection, parasitism, vanishing species, and evolution.

The book seems especially intended for schools that are poorly equipped for laboratory work and in which book work and recitation work are prominent, and laboratory work is small in amount and superficial in character. The strong points in the book are the economic and natural history features, and the weak points are the trifling amount of laboratory work outlined and the uncorrelated and more or less antiquated physiological matter. One still meets with a liver in the crayfish and the fresh water mussel and with organs of hearing in the jellyfish and the crayfish. The statement is made that if an earthworm "be cut in two near the middle each half will often develop into a complete worm." The observations to the contrary by Morgan and others were apparently unknown to the author.

In the opinion of the reviewer, the book contains much useful matter that might well find its way into the secondary school course in zoology, especially in the agricultural districts. It should, however, be made to supplement a good laboratory and field course rather than to supplant it.

FRANK SMITH.